Name..
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Math 1210-2
Quiz 5 Solutions
October 5, 2007

Show all work for complete credit!

1) Consider the graph of the equation

$$x^2 + 3\,x\,y + y^2 = -1$$

1a) Show that the point (2,-1) is on the graph

(1 points)

For a point (x,y) to be on the graph it must satisfy the equation. So, plug (x,y)=(2,-1) into the equation!

$$2^2 + 3\,(2)\,(-1) + (-1)^2 = -1\ ?$$
$$-1 = -1$$

is true.

1b) Use implicit differentiation to find the slope y′ of the graph above, at the point (2,-1).

(3 points)

We think of y representing an (unknown) function of x, y=f(x), and use the chain rule to differentiate both sides of the equation:

$$2\,x + 3\,y + 3\,x\left[\frac{dy}{dx}\right] + 2\,y\left[\frac{dy}{dx}\right] = 0.$$

Since x=2 and y=-1, we get

$$4 - 3 + (6 - 2)\left[\frac{dy}{dx}\right] = 0$$
$$\left[\frac{dy}{dx}\right] = \frac{-1}{4}$$

2) The bottom of a ladder is sliding a away from a wall along (wet) level pavement, while the top slides down the wall. The ladder is 13 feet long. At the instant when the bottom of the ladder is 5 feet from the wall it is sliding away at a rate of 2 feet/second. How fast is the top of the ladder sliding down at that instant?

(6 points)

The picture to draw is a right triangle with base "x" = the distance from the wall to the bottom of the ladder, and height y from ground level along the wall, to the top of the ladder. We are told that dx/dt= 2 feet/second at the moment of interest. We wish to find dy/dt at that instant. Since the ladder is 13 feet long and $13^2 = 169$, the equation relating x and y is the Pythagorean Theorem:

$$x^2 + y^2 = 169.$$

Taking the time derivative of this yields

$$2\,x\left[\frac{dx}{dt}\right] + 2\,y\left[\frac{dy}{dt}\right] = 0.$$

At the moment of interest x=5, so $y = \sqrt{169 - 25}$=12. *Plugging into the equation above and dividing by 2* *yields*

$$5\,(2) + 12\left[\frac{dy}{dt}\right] = 0$$

$$\left[\frac{dy}{dt}\right] = \frac{-5}{6}$$

So, to be precise, the ladder is falling at a rate of $\frac{5}{6}$ *feet/second.*