Name.....

UID.....

Math 1210-2

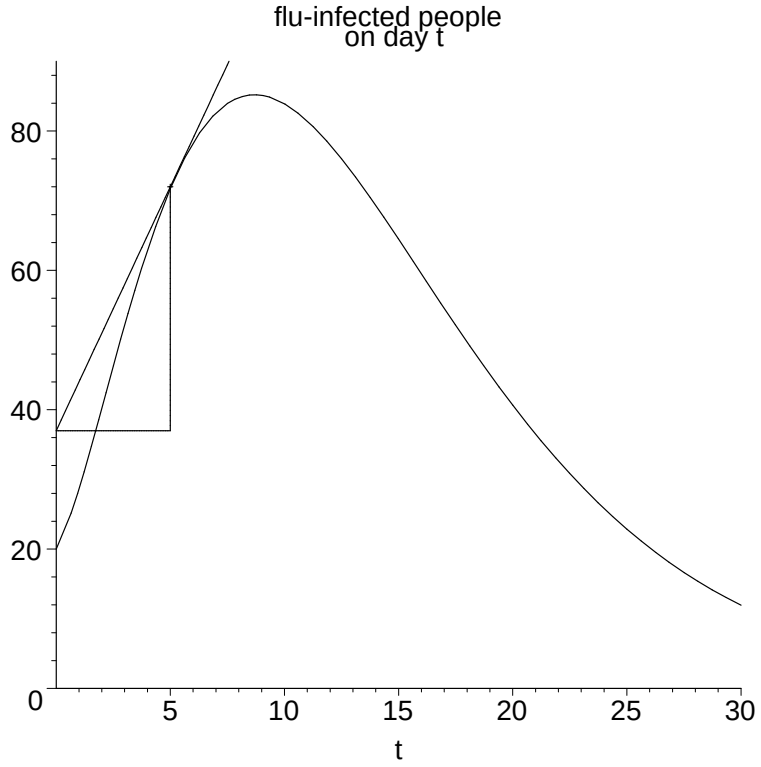
Quiz 4 Solutions

September 28, 2007

Show all work for complete credit!

1) Mathville is hit with a flu epidemic. The number of sick people at time t days after the illness hits is plotted below.

For similar problems, see lecture notes and text discussion and problems of section 2.1. You also have webworks problems like this, but with function formulas given.



1a) About how many people were sick with the flu on day 5?

(1 point)

It appears that the point (5,72) is almost on the graph, so P(5) is about 72.

1b) At about what rate was the number of sick people increasing on day 5?

(2 points)

(instantaneous) rate of change is the limit of average rates of change, which corresponds to slopes of secant lines on the graph, where the units of rise/run are people/day. We can approximate the rate of change of the number of sick people, P(t), either by drawing a secant line through the point t=5 and a nearby one, or the tangent line at t=5, and then estimating its slope (in units of people per day). The

sketch and triangle above shows that the slope is about (72-38)/5, about 9 people per day. Given the fact that you didn't have rulers (although paper edge can work), if you used a method like this and came up with a rate between 6 and 12 people per day, you did well.

2) Find the following derivatives

Sections 2.3-2.5, webworks:

2a) $D_t \ (2t+1)^5 \cos(3t)$.

(2 points)

Use the product rule and the chain rule:

$$D_t \ (2t+1)^5 \cos(3t) = 10\,(2t+1)^4 \cos(3t) - 3\,(2t+1)^5 \sin(3t)$$

Section 2.4, notes, webworks:

2b) $f'(\pi)$, for $f(x) = \dfrac{\tan(x)-5}{\sec(x)}$. (You should evaluate all trig functions in your final answer.)

(2 points)

note that

$$\frac{\tan(x)-5}{\sec(x)} = \frac{\left[\dfrac{\sin(x)}{\cos(x)} - 5\right]}{\left[\dfrac{1}{\cos(x)}\right]} = \sin(x) - 5\cos(x).$$

Thus

$$f'(x) = \cos(x) + 5\sin(x).$$

So

$$f'(\pi) = -1.$$

3) Use the limit definition of derivative to compute ONE of the following derivatives (your choice, but if you try both, indicate which one you want graded!)
Section 2.2, 2.4, and accompanying lecture notes, and text problems.

3a) $D_x \left(\dfrac{1}{x^2}\right)$.

3b) $D_x \sin(x)$. (If you choose this one you may use the trig limits we used in class and in the previous webworks assignment.)

(3 points).

3a)

$$\lim_{h\to 0} \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \lim_{h\to 0} \left[\frac{1}{h}\right]\left(\frac{1}{(x+h)^2} - \frac{1}{x^2}\right)$$

$$= \lim_{h\to 0} \frac{x^2-(x+h)^2}{h\,x^2\,(x+h)^2} = \lim_{h\to 0} \frac{-2\,h\,x - h^2}{h\,x^2\,(x+h)^2}$$

$$= \lim_{h \to 0} \frac{-2x - h}{x^2 (x+h)^2} \quad = -\frac{2x}{[x^4]} \quad = -\frac{2}{x^3}$$

Of course, you can check your answer with the power rule.

3b)

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} \quad = \lim_{h \to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)(\cos(h) - 1)}{h} + \frac{\cos(x)\sin(h)}{h}$$

$$= \sin(x)\,0 + \cos(x)\,1 \quad = \cos(x)$$

which is hopefully one of the derivative formulas you've memorized.