

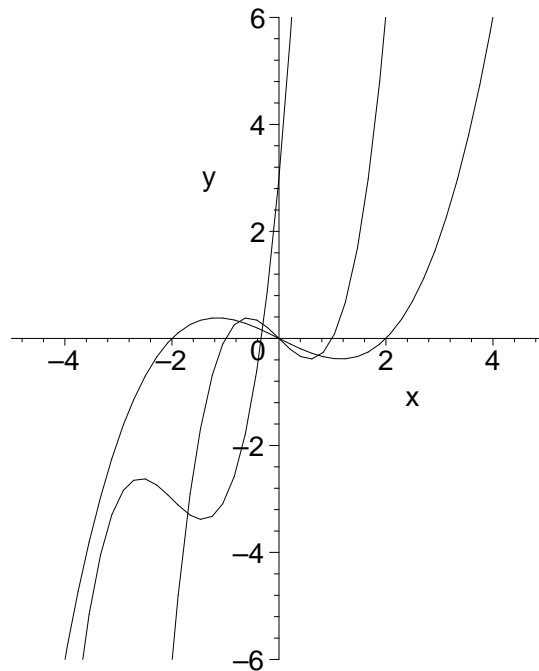
Name.....

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Math 1210-2
Quiz 3 Solutions
 September 7, 2007

Show all work for complete credit!

1) Here is the graph of the function $f(x) = x^3 - x$:



1a) Precisely how is the graph of $g(x) = (x+2)^3 - x - 5$ obtained by translating the graph of $f(x)$ pictured above? Describe the translation(s) precisely, and then add the sketch the graph $y = g(x)$ into the picture above.

(3 points)

$g(x) = (x+2)^3 - (x+2) - 3 = f(x+2) - 3$. Thus the graph of $y=f(x)$ is shifted two units to the left, and three units down. So the graph of $y=g(x)$ looks like $y=x^3$, but the point which used to be $(0,0)$ got shifted to $(-2,-3)$, see picture above.

1b) Precisely how is the graph of $h(x) = \left[\frac{x}{2}\right]^3 - \frac{x}{2}$ obtained from the graph of $f(x)$? Describe the scaling precisely, and add the graph of $h(x)$ to the picture above.

(3 points)

$h(x) = f(x/2)$, so the graph of $y=h(x)$ is obtained from the graph of f by **STRETCHING** by a factor of 2 in the x -direction! Thus the point $(1,0)$ on the graph $y=x^3-x$ corresponds to the point $(2,0)$ on the graph $y=h(x)$.

2) $p(x) = \sqrt{x}$ and $f(x) = x^3 - x$.

2a) What is the formula for the product function $(fp)(x)$?

(1 point)

$$\begin{aligned} f(x) p(x) &= (x^3 - x) \sqrt{x} \\ &= x^{(7/2)} - \sqrt{x} \end{aligned}$$

2b) What is the formula for the composition $(f \circ p)(x) = f(p(x))$?

(1 point)

$$\begin{aligned} f(p(x)) &= f(\sqrt{x}) \\ &= x^{(3/2)} - \sqrt{x} . \end{aligned}$$

2c) What is the domain of $(f \circ p)$?

(1 point)

Since only non-negative numbers have real square roots, the domain of $(f \circ p)$ is $0 \leq x$.

2d)* What is the domain of $(p \circ f)$? Hint: first write the formula for $(p \circ f)$, then look at the graph of f in problem 1 for inspiration.

(1 point)

$$p(f(x)) = \sqrt{x^3 - x}$$

so we must have $0 \leq x^3 - x$. Looking at the graph $y = x^3 - x$ on one page 1 it appears that this happens if $-1 \leq x \leq 0$ or $1 \leq x$. If this is your answer you get full credit, even if you don't justify it algebraically.

The algebraic justification for this answer is that

$$x^3 - x = x(x^2 - 1)$$

and if we want this expression to be non-negative either both terms must be greater than zero, or both terms must be less than or equal to zero. The only way both terms can be greater than or equal to zero is if $1 \leq x$. The only way both terms can be less than or equal to zero is if $x \leq 0$ and $-1 \leq x$.