Name...
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Math 1210-2

Quiz 2 Solutions

August 31, 2007

Show all work for complete credit! Every question below has something to do with the graph $y = x^2 + 3x$.

1) Use the limit definition of derivative to compute $f'(x)$, for $f(x) = x^2 + 3x$.

(3 points)

$$f(x+h) = x^2 + 2xh + h^2 + 3x + 3h$$

$$f(x) = x^2 + 3x$$

so,

$$\frac{f(x+h) - f(x)}{h} = \frac{[2xh + h^2 + 3h]}{h}$$

$$= 2x + h + 3.$$

As h approaches zero the limit is $f'(x) = 2x + 3$.

2a) What is the slope of the parabola $y = x^2 + 3x$ at the point **P** = (-2,-2)?

(1 point)

Since $f'(x) = 2x + 3$ *is the slope function, when x=-2 the slope of the graph at* ***P*** *is* $f'(-2) = -1$.

2b) What is the slope-intercept equation of the line through **P**, with the slope you computed in (2a)? (This line is the tangent line to the parabola at **P**.)

(2 points)

The point-slope equation is

$$y + 2 = -(x+2)$$

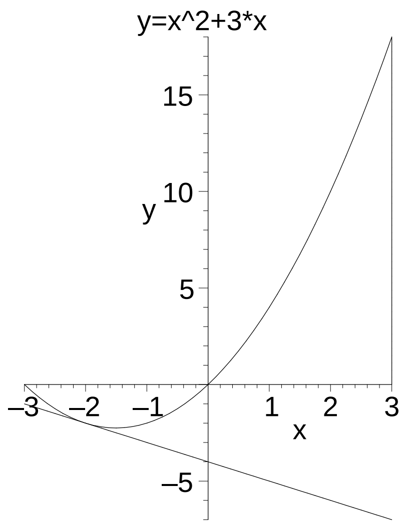
which yields slope-intercept equation

$$y = -x - 4.$$

2c) Draw the tangent line from (2b) onto the picture below, so that it passes through **P** and has the correct y-intercept. Your slope may not "look" correct, because the scales are different in the x and y-directions.

(1 point)

The y-intercept of the tangent line is -4, the slope is -1, and it passes through (-2,-2).



3a) Sketch the region under the graph of $y = x^2 + 3\,x$ (and above the x-axis), between x=0 and x=3, in the picture above.

(1 point)

I can't get MAPLE to shade the triangular region above, with base the segment from x=0 to x=3 on the x-axis, height of 18 (up to the point (3,18) on the graph of f), and "hypotenuse" the curved graph, for x from 0 to 3.

3b) Find the area of the region you sketched in (3a).

(2 points)

Definite integrals of non-negative functions yield the area under the graph, so

$$A = \int_0^3 x^2 + 3\,x\,dx$$

An antiderivative of f(x) is

$$\mathrm{F}(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2$$

so the area is

$$\mathrm{F}(3) - \mathrm{F}(0) = 9 + \frac{27}{2} - 0$$

$$= \frac{45}{2}.$$