

Name _____

Student I.D. _____

Math 1210-2
Practice Exam #1 **Solutions**
September 13, 2007

Please show all work for full credit. This exam is closed book, closed note, closed calculator. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful to not spend too long on any one problem. Good Luck!!

Score

POSSIBLE

1 _____ 20

2 _____ 20

3 _____ 30

4 _____ 20

5 _____ 10

TOTAL _____

1) Shown below is the graph of $y=\cos(x)$. Add the following sketches to the picture. For parts (b),(c), explain in words how the graphs are related to the original graph of $y=\cos(x)$.

1a) $y = \sin(x)$.

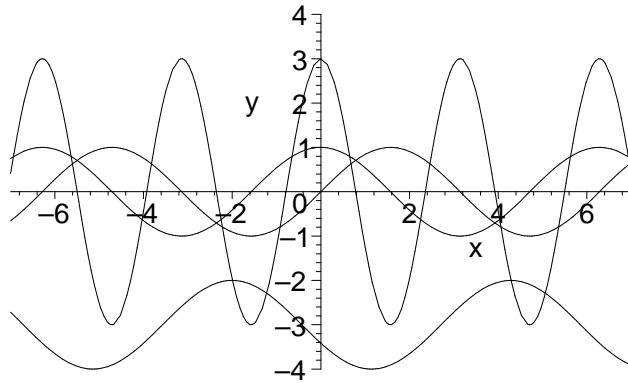
($y=\sin(x)$ looks like $y=\cos(x)$, only translated over, see 1d)

1b). $y = \cos(x + 2) - 3$

this is the graph of $y=\cos(x)$ translated 2 units to the left and down 3 units...it's the lowest curve below

1c) $y = 3 \cos(2x)$.

this is obtained from $y=\cos(x)$ by stretching vertically by a factor of 3 and shrinking horizontally by a factor of 2. So, for example, the period of this function is π , rather than 2π . It's the curve below with amplitude 3.



1d) Use the addition angle formula for $\cos(\alpha + \beta)$ to show that the graph of $y = \sin(x)$ (that you sketched

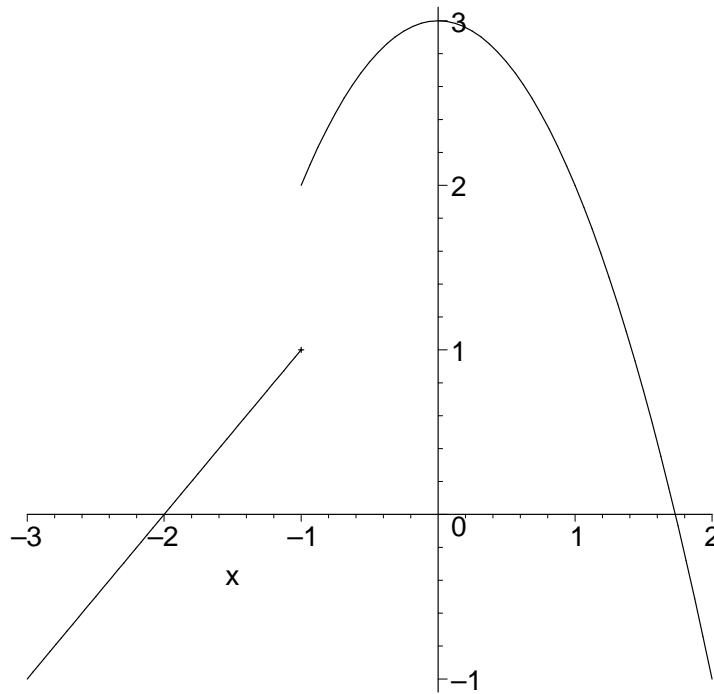
in part (a)) is a translation of the graph $y = \cos(x)$ by $\frac{\pi}{2}$ to the right.

The translation of $y=\cos(x)$ by $\frac{\pi}{2}$ to the right has equation $y = \cos\left(x - \frac{\pi}{2}\right)$ But

$$\cos\left(x - \frac{\pi}{2}\right) = \cos(x) \cos\left(\frac{\pi}{2}\right) - \sin(x) \sin\left(\frac{-\pi}{2}\right)$$

Since $\cos\left(\frac{\pi}{2}\right) = 0$ and $\sin\left(\frac{-\pi}{2}\right) = -1$ we deduce our graph is $y = \cos\left(x - \frac{\pi}{2}\right) = \sin(x)$.

2) Here the graph of a function $y=f(x)$:



Use the graph to deduce the following limits. If the limit does not exist, write DNE.

2a)

$$\lim_{x \rightarrow (-1)^+} f(x)$$

(5 points)

As x approaches -1 from the right the height of the graph $y=f(x)$ approaches 2 , so the value of the limit is 2

2b)

$$\lim_{x \rightarrow (-1)^-} f(x)$$

(5 points)

As x approaches -1 from the left the height of the graph approaches 1 , so the value of the limit is 1 .

2c)

$$\lim_{x \rightarrow (-1)} f(x)$$

(5 points)

The limit DNE because the values of $f(x)$ do not approach a single number as x approaches 1 .

2d)

$$\lim_{x \rightarrow (-2)} f(x)$$

(5 points)

The height of the graph approaches zero as x approaches -2 , so the value of the limit is 0 .

3a) Use the limit definition of derivative to find the slope function for the graph $y = x^3 + 2x$. (15 points)

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^3 + 2(x+h) - x^3 - 2x}{h}$$

We use

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

and simplify, to obtain

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3 + 2h}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 + 2 \\ &= 3x^2 + 2. \end{aligned}$$

The slope function, i.e. the derivative of $f(x)$ is $3x^2 + 2$.

3b) Check your work in part (3a), using the rules we learned for differentiating polynomials. (5 points)

$$(x^3 + 2x)' = 3x^2 + 2.$$

3c) Find the equation of the tangent line to the graph $y = x^3 + 2x$, at the point on the graph with x-coordinate 1. (5 points)

when $x=1$ the y-coordinate on the graph is 3, so the tangent line passes through (1,3). Its slope is given by the slope function $f'(x) = 3x^2 + 2$, which has value 5 when $x=1$. Thus the (point-slope) equation for the tangent line is

$$y - 3 = 5(x - 1).$$

3d) Where does the line you found in (3c) intercept the x-axis? Where does it intercept the y-axis? (5 points)

The line intersects the x-axis when $y=0$, so we solve

$$-3 = 5(x - 1),$$

obtaining $x=2/5$. The line intersects the y-axis when $x=0$, which yields $y=-2$.

4) A speedometer of a car moving along a road has recorded car's (positive) velocity in miles per hour, from time 0 to 1 hour, and the data fits the following function:

$$v(t) = 60 + 2t - \frac{1}{30}t^2.$$

4a) How far does the car travel in that hour?

(10 points)

$$\begin{aligned} s(1) - s(0) &= \int_0^1 v(t) dt \\ &= \int_0^1 \left(60 + 2t - \frac{1}{30}t^2 \right) dt \end{aligned}$$

Since an antiderivative of the velocity function is $F(t) = 60t + t^2 - \frac{1}{90}t^3$ the value of the definite integral is $F(1) - F(0) = 60\frac{89}{90}$ miles.

4b) When is the car traveling the fastest? And what is its velocity then? (Hint: its acceleration will be zero at that instant!)

(10 points)

O.K., I thought this would be a great practice exam question, but probably not one to put on the actual exam. And then, I confused minutes with hours so that the answer is not what I expected! But, here goes:

The velocity will be maximized when the acceleration is zero, So we compute $v'(t)$:

$$a = 2 - \frac{t}{15}.$$

Now, this shows that for t between zero and one hours the acceleration is always positive, so the car is speeding up. Thus for our time interval, the maximum velocity is at $t=1$ hour, and is $v(1) = 61\frac{29}{30}$ miles per hour. If the car kept going for 30 hours with the same velocity function, then the acceleration would be zero after 30 hours, and the car's velocity then would be $v(30)=90$ miles per hour.

5) For $f(x) = x^2 + 1$ and $g(y) = \sqrt{y}$, consider x 's and y 's which lie in the open intervals of points satisfying

$$|x - 1| < .1, \quad |y - 4| < .1$$

5a) What are the best estimates you can make for the values of $f(x)g(y)$ based on this information? (You don't need to simplify your answer, which should be an open interval containing the number $4=f(1)g(4)$.)

(5 points)

A product of positive numbers is maximized when each factor is as large as possible, and minimized when each factor is as small as possible. The x values and y values satisfy

$$0.9 < x < 1.1, \quad 3.9 < y < 4.1$$

So we also know (since $f(x)$ and $g(y)$ increase as x and y increase)

$$1.81 < f(x) < 2.21, \quad \sqrt{3.9} < g(y) < \sqrt{4.1}$$

thus

$$1.81\sqrt{3.99} < f(x)g(y) < 2.21\sqrt{4.1}.$$

5b) What are the best estimates you can make for $f(x)/g(y)$? (Your answer should be an open interval containing $1=f(1)/g(4)$.)

(5 points)

A quotient of two positive numbers is maximized when the numerator is as large as possible and the denominator is as small as possible. The quotient is minimized when the numerator is as small as possible, and the denominator is as large as possible. Thus

$$\frac{1.81}{\sqrt{4.1}} < \frac{f(x)}{g(y)} < \frac{2.21}{\sqrt{3.9}}.$$