Math 1210-2
Wednesday 10/2

Friday quiz topics
§2.7-2.8
implicit differentiation & related rates

§2.9 Differentials and approximation

these will be very useful in chapters 4-5, for integration.
for now we use differentials to do tangent line approximation and to get used to the notation

Recall, if $y = f(x)$

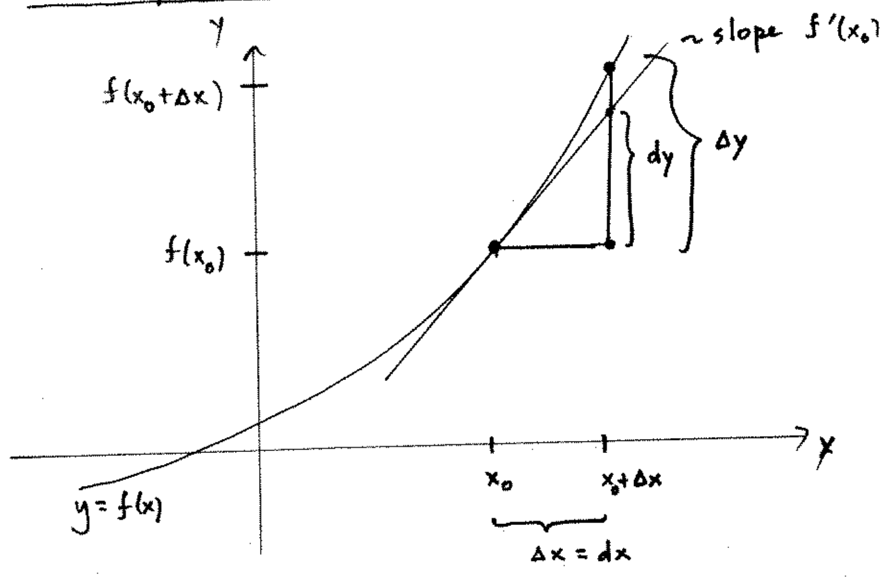
$f'(x) = \frac{dy}{dx}$ are two of our notations for derivative

$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

The idea of differentials is to focus on the linear approximation to f

Set $dx = \Delta x$
define $dy = f'(x)\,dx$.
dx and dy are called differentials

This is the tangent line approximation to the exact change in y, i.e. $\Delta y = f(x+\Delta x) - f(x)$



$\Delta y = f(x_0+\Delta x) - f(x_0)$
$dy = f'(x_0)\,dx$

Notice $\frac{dy}{dx} = \frac{f'(x)\cancel{dx}}{\cancel{dx}} = f'(x) = \frac{dy}{dx}$, i.e. we can think of the derivative $\frac{dy}{dx}$ as a quotient of two differentials dy, dx.

amazingly, this hocus pocus turns out to be useful.

Exercise 1 : Use $dy = f'(x)dx$ to find dy:

1a) $y = \sqrt{x^2+3x}$

1b) $y = x(x^2-1)^{-2}$

1c) $y = \sec(x^3)$

Approximation:

Exercise 2 Using $y = \sqrt{x}$ and differentials, approximate (and then compare to "exact" calculator values

2a) $\sqrt{4.2}$ $x = 4$
$dx = .2$

2.04939...

2b) $\sqrt{8.7}$

≈ 2.94958

<u>Exercise 3</u> Use differentials to estimate the increase in surface area of a soap bubble when its radius increases from 3 to 3.025 inches.

$$A = 4\pi r^2$$

Exact change
$4\pi(3.025^2 - 3^2)$
≈ 1.893 in^2

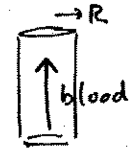
<u>Error analysis</u>

<u>Exercise 4</u>: The side lengths of a cube are measured to be 10 cm $\pm$ .1 cm
Use differentials to estimate how close the volume is to 10^3 cm^3.

$$V = x^3$$

Exact error est:
$(10.1)^3 = 1030.301$
$(9.9)^3 = 970.299$

<u>Exercise 5</u>: Poiseuille's Law (physics) for blood flow says the flux (Volume/time) of blood flowing through a cylindrical artery is proportional to the 4th power of the radius, i.e.



$$F = kR^4$$ (k depends on blood pressure)

If the radius decreases by 5%, estimate the % decrease in blood flow (assuming constant blood pressure).

Exact % dec:
≈ 18.6 %