Math 1210-2

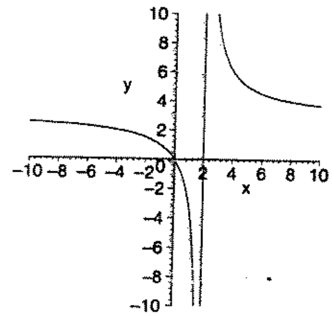
Wednesday Oct 24

§3.5 Graphing with Calculus

§1.5 Limits involving infinity

<u>Exercise 1</u>: Use calculus to plot the graph of $y = \underset{f(x)}{\overset{\shortparallel}{\frac{3x}{x-2}}}$. Also use the limit behavior to discuss asymptotes

(See page 2)



domain:

$\lim_{x \to \infty} f(x) =$

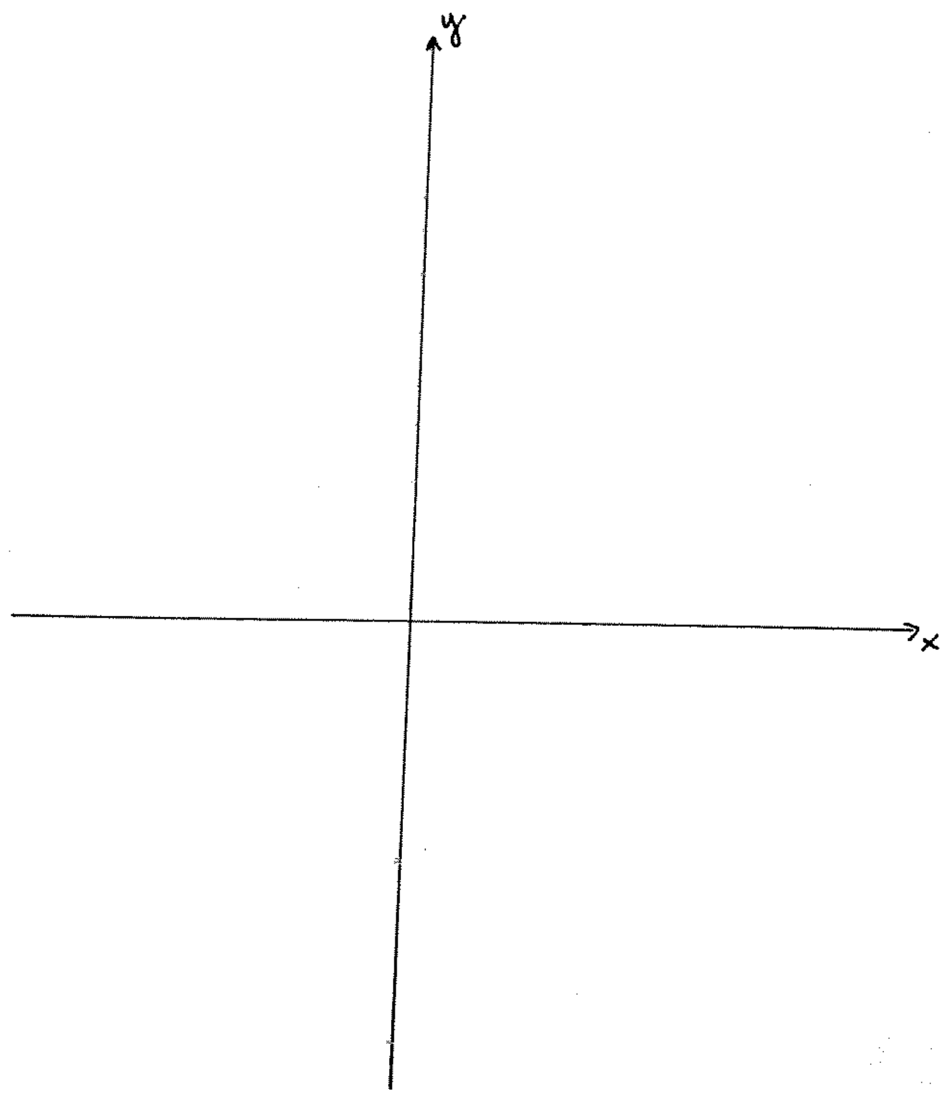
$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$f'(x)$

$f''(x)$



Definition A $\lim_{x \to \infty} f(x) = L$ means imprecisely: As x approches infinity the values of $f(x)$ approach L

precisely: $\forall \varepsilon > 0 \ \exists N$ s.t. $x > N \Rightarrow |f(x) - L| < \varepsilon$

"for all $\varepsilon > 0$ there exists N s.t. $x > N$ implies $|f(x) - L| < \varepsilon$"

$\lim_{x \to -\infty} f(x) = L$ defined analogously.

Also,

$\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = -\infty$.

Definition B $\lim_{x \to c^+} f(x) = +\infty$ means imprecisely: As x approches c from the right the values of $f(x)$ approach infinity.

precisely: $\forall N > 0 \ \exists \delta$ s.t. $c < x < c + \delta \Rightarrow f(x) > N$

$$\left.\begin{array}{l} \lim_{x \to c^+} f(x) = -\infty \\ \lim_{x \to c^-} f(x) = +\infty \\ \lim_{x \to c^-} f(x) = -\infty \end{array}\right\} \text{defined analogously.}$$

Definition C A line $ax + by + c = 0$ is an <u>asymptote</u> to the graph $y = f(x)$ if some points on the graph become arbitrarily close to the line, as the distance to the origin, $\sqrt{x^2 + y^2}$, becomes arbitrarily large.

precisely,

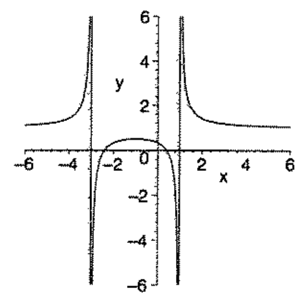
If $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$, then <u>$y = L$</u> is a <u>horizontal asymptote</u>

If $\lim_{x \to c^+} f(x)$, or $\lim_{x \to c^-} f(x)$ is $+\infty$ or $-\infty$, then <u>$x = c$</u> is a <u>vertical asymptote</u>

If $\lim_{x \to \infty} \left[f(x) - (mx + b) \right] = 0$ (or if $\lim_{x \to -\infty} (\text{〃}) = 0$)

then <u>$y = mx + b$</u> is a <u>diagonal asymptote</u>

<u>Exercise 2</u> graph $y = \frac{2}{x^2+2x-3} + 1$ using Calculus and limits (but maybe not f'')



<u>Exercise 3</u> graph $y = \frac{-2x^2-5x+1}{x+3}$ using Calculus.

Hint: First do long division.