Math 1210-2
Tuesday Oct 16

§3.3 Local extrema (as opposed to global extrema)

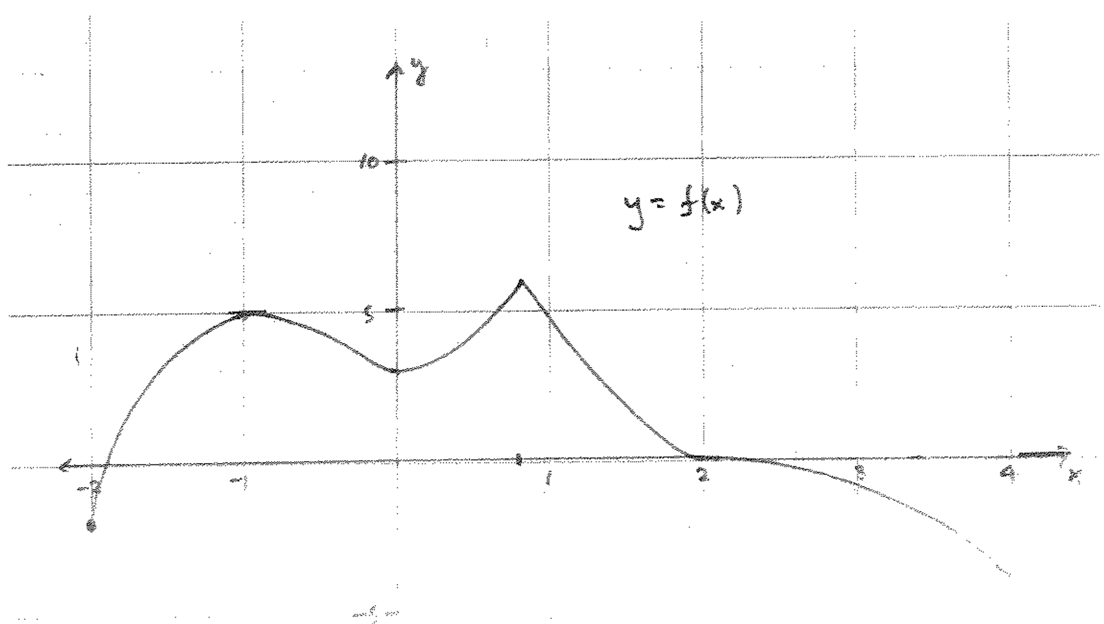
Definition:
Let f have domain S, with $c \in S$.
($\in$: an element of)

Then
(i) $f(c)$ is a local maximum value of f if there is an interval $(\tilde{a}, \tilde{b})$ containing c so that $f(c)$ is the maximum value of f on $(\tilde{a}, \tilde{b}) \cap S$. ($\cap$: intersection)

(ii) $f(c)$ is a local minimum value of f if there is an interval $(\tilde{a}, \tilde{b})$ containing c so that $f(c)$ is the minimum value of f on $(\tilde{a}, \tilde{b}) \cap S$.

(iii) $f(c)$ is a local extreme value of f if it is either a local maximum or a local minimum value.

Exercise 1: Here is a graph of $y = f(x)$, on the interval $[-2, 4]$. Identify all local and global extreme values. (Note, f is continuous on $[-2, 4]$, so it attains a global max & min value.)



Where do local extrema occur?

If $f(c)$ is a local extreme value with c interior to the domain S, then (as for interior global extrema), c is either stationary ($f'(c)=0$) (ii) or singular ($f'(c)$ DNE) (iii)

Otherwise c is an endpoint (i)

Exercise 2 The reasoning above shows that every local extreme value occurs at a critical point ((i), (ii) or (iii)). Is the converse statement true? That is, is there a local extreme value at every critical point?

Exercise 3 Can someone explain why each of the following tests is true, based on our previous increasing/decreasing, CU, CD discussions?

Theorem A First Derivative Test

Let f be continuous on the open interval (a,b) that contains a critical point c.

a c b

(i) If sign f': $+++$ on (a, c), $---$ on (c, b), then $f(c)$ is local max value

(ii) If sign f': $---$ on (a, c), $+++$ on (c, b), then $f(c)$ local min value

(iii) If sign f': $+++$ on (a, c), $+++$ on (c, b) or $---$ on (a, c), $---$ on (c, b), then $f(c)$ is neither local max or local min

Theorem B Second derivative test: Let f', f'' exist at each point in interval (a,b) containing critical point c (so $f'(c)=0$).

(i) If $f''(c) < 0$ then $f(c)$ is local max val.

(ii) If $f''(c) > 0$ then $f(c)$ is local max val.

Exercise 4 Find all local extrema of $f(x) = x^2 - 6x + 5$, and sketch the graph.

Exercise 5 Find all local extrema of $g(x) = x^4 - 2x^3$, and sketch the graph