§3.3 Local extrema (as opposed to global extrema)

Definition:
Let $f$ have domain $S$, with $c \in S$.

Then
(i) $f(c)$ is a local maximum value of $f$ if there is an interval $(a, b)$ containing $c$ so that $f(c)$ is the maximum value of $f$ on $(a, b) \cap S$.

(ii) $f(c)$ is a local minimum value of $f$ if there is an interval $(a, b)$ containing $c$ so that $f(c)$ is the minimum value of $f$ on $(a, b) \cap S$.

(iii) $f(c)$ is a local extreme value of $f$ if it is either a local maximum or a local minimum value.

Exercise 1: Here is a graph of $y = f(x)$, on the interval $[-2, 4]$. Identify all local and global extreme values. (Note, $f$ is continuous on $[-2, 4]$, so it attains a global max & min value.)
Where do local extrema occur?

If \( f(c) \) is a local extreme value with \( c \) interior to the domain \( S \),
then (as for interval global extrema), \( c \) is either stationary or singular.
\[ f'(c) = 0 \quad f''(c) \text{ DNE} \]
Otherwise \( c \) is an endpoint

Exercise 2 The reasoning above shows that every local extreme value occurs at
a critical point \( (i), (ii), (iii) \). Is the converse statement true?
That is, is there a local extreme value at every critical point?

Exercise 3 Can someone explain why each of the following tests is true, based
on our previous increasing/decreasing, CV, CD discussions?

Theorem A First Derivative Test
Let \( f \) be continuous on the open interval \((a, b)\) that contains a critical point \( c \).

(i) If \[ \text{sign} \ f' = \frac{a}{c} \quad \frac{+}{-} \quad \frac{+}{-} \quad \frac{c}{b} \]
then \( f(c) \) is local max value \( c \)

(ii) If \[ \text{sign} \ f' = \frac{a}{c} \quad \frac{-}{+} \quad \frac{+}{-} \quad \frac{c}{b} \]
then \( f(c) \) local min value \( c \)

(iii) If \[ \text{sign} \ f' = \frac{a}{c} \quad \frac{+}{+} \quad \frac{+}{+} \quad \frac{c}{b} \]
or \[ \frac{a}{c} \quad \frac{-}{-} \quad \frac{c}{b} \]
then \( f(c) \) is neither local max \( c \) nor local min \( c \)

Theorem B Second derivative test: Let \( f' \) \( f'' \) exist at each point in interval \((a, b)\)
containing critical point \( c \) \( (so \ f'(c) \text{ DNE}) \)

(i) If \( f''(c) < 0 \) then \( f(c) \) is local max value \( c \)

(ii) If \( f''(c) > 0 \) then \( f(c) \) is local min value \( c \)
Exercise 4: Find all local extrema of \( f(x) = x^2 - 6x + 5 \) and sketch the graph.

Exercise 5: Find all local extrema of \( g(x) = x^4 - 2x^3 \), and sketch the graph.