Math 1210-2
Monday Oct 15 §3.1-3.2

We are discussing pages 3-4 of Fri Oct 5 notes,
in particular Theorems A & B about finding the maximum and minimum (extreme) values of continuous functions defined on closed intervals.
After we finish that discussion & examples, begin

§3.2 Monotonicity and concavity

Definition Let f be defined on an interval I (open, closed, or neither). We say
(i) f is increasing on I if for every pair of numbers x_1, x_2 in I,

$$x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$$ (graph rises to the right)

(ii) f is decreasing on I if for every pair of numbers $x_1, x_2 \in I$,

$$x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$$ (graph sinks to the right).

(iii) f is strictly monotonic on I if it is either increasing on I, or decreasing on I

Exercise 1 Using 2a), 2b) from last Friday's notes, find (largest possible) intervals on which $f(x)$ and $g(x)$ are strictly monotone

Theorem A : If f is continuous on I and differentiable at each interior point of I (not an endpoint)
then
(i) $f'(x) > 0 \ \forall$ interior pts $x \Rightarrow f$ increasing on I
(ii) $f'(x) < 0 \ \forall$ interior pts $x \Rightarrow f$ decreasing on I

Definition : f is <u>concave up</u> on I if (the slope) f' is increasing on I

f is <u>concave down</u> on I if f' is decreasing on I


applying Theorem A to $f'(x)$ we deduce

Theorem B Let f be twice differentiable on I. Then
(i) $f''(x) > 0 \ \forall$ interior points $\Rightarrow f$ is concave up (CU) on I
(ii) $f''(x) < 0$ " " $\Rightarrow$ " concave down (CD) on I

Exercise 2 : Consider $V(x) = x(6-2x)^2 = 4(x^3 - 6x^2 + 9x)$, $-\infty < x < \infty$.
(which we studied for $0 \le x \le 3$, for our box problem last Friday).

Use $V'(x)$ and $V''(x)$ to find where V is increasing, decreasing, CU, CD.
Then sketch the graph!