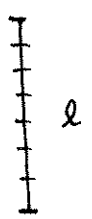
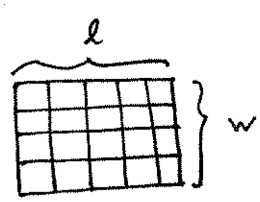
Math 1210-2
Monday Oct 1
§2.8 related rates
+
geometry review

- Do exercise 3,4 on last page of Fri. Sept 28 notes.

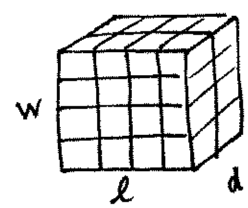
Quick review of geometry measurements - important in <u>related rates</u> problems, also in Chapter 3 for <u>optimization</u> problems



length
$L = \ell$ units
e.g. cm^1
ft, etc.

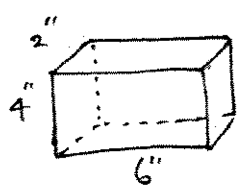


area
$A = \ell w$ square units
e.g. cm^2
ft^2



volume
$V = \ell w d$ cubic units
e.g. cm^3
ft^3

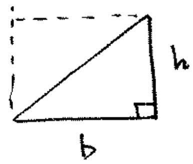
<u>Exercise 1</u> : A rectangular block has dimensions 2 in, 4 in, 6 in.



1a) Find the volume of the block

1b) Find the total surface area of all 6 faces

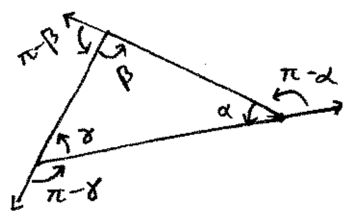
1c) Find the total edge length, of all 12 edges.



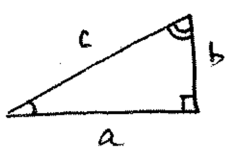
$A = \frac{1}{2} bh$, right triangle

(half a rectangle)

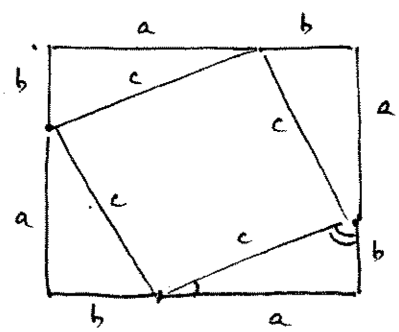
Exercise 2 : Explain why the interior angles of any triangle add up to π radians



Exercise 3 Explain the Pythagorean theorem, which says that the hypotenuse length c of a right Δ is $\sqrt{a^2+b^2}$, where a, b are the "leg" lengths. This is really an area computation! Compute the square area below two ways, using Exercise 2. to show the interior quadrilateral is a square



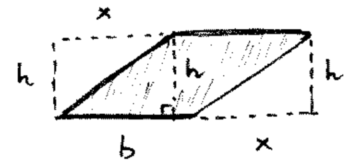
$a^2+b^2=c^2$



Exercise 4 Explain why

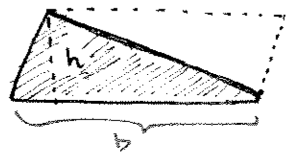
4a) the area of a parallelogram is the length b of the base, times the height h,

$$A = bh$$



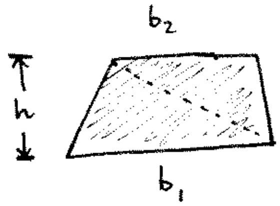
4b) the area of any triangle is

$$A = \tfrac{1}{2}bh$$



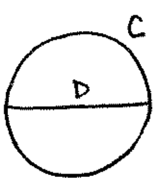
4c) the area of a trapezoid is

$$A = \left(\frac{b_1+b_2}{2}\right)h$$



→ notice how all area formulas give square units.

Circles and disks

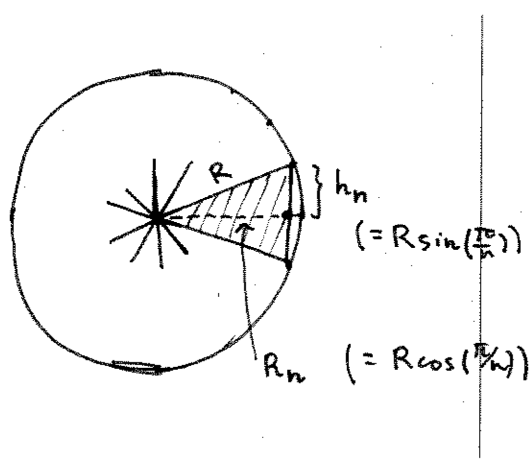


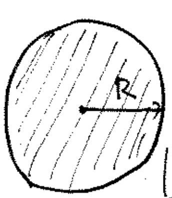
π is defined as $\frac{C}{D}$, C = circumfrence, D = diameter. This ratio is independent of the size of the circle

So since $D = 2R$,

$$\boxed{C = 2\pi R}$$

Exercise 5 Use the circumfrence formula, subdivide the radius R disk into n congruent sectors, compute the area of the inscribed triangles, let $n \to \infty$ to deduce

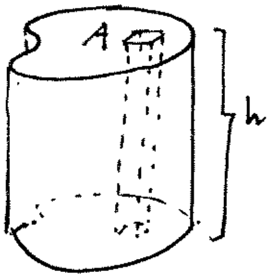




$$\boxed{A = \pi R^2}$$

→ square units for area, units for circumfrence.

3-dimensional objects



right-vertical cylinder with all horizontal cross-sections identical, has volume

$$V = Ah$$

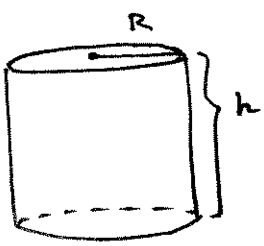
(A = cross-section area).

Reason:

Exercise 6 Consider a solid cylinder with circular cross sections, radius R; and height h.

6a) What is volume V?

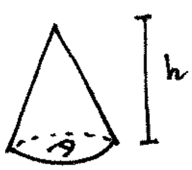
6b) What is the total surface area? (of its outside)



Balls, spheres, cones:

solid radius R ball has volume $V = \frac{4}{3}\pi R^3$ cubic units!

the sphere shell has area $A = 4\pi R^2$ square units!



solid cone with base area A and height h has volume

$$V = \frac{1}{3}Ah \quad \text{cubic units}$$

↑
?!

We'll see how to derive these formulas using Calculus, later on.