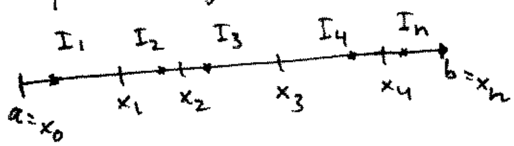
Math 1210-2
Wednesday Nov. 7

- Carnival signup! (actual event is <u>next</u> Wed.)
- Tonight, 7-8:30 p.m., using MAPLE to do Calculus
  LCB 115. (a computer classroom @ 30 terminals)

§4.2 The definite integral. (Motivated by yesterday's discussion of "area")

let $f(x)$ have domain $[a,b]$
The definite integral of f, from $x=a$ to $x=b$ will be defined as a limit of <u>Riemann sums</u>:

Let P be a partition of $[a,b]$ into n subintervals:

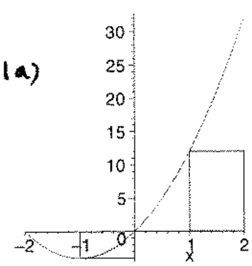


$I_i=[x_{i-1},x_i]$, $\Delta x_i = \text{width} = x_i - x_{i-1}$
In each subinterval I_i, pick a sample point $\bar{x}_i$ (it could be x_{i-1}, x_i, or any point in between)

Then, for this partition and choice of sample points, the <u>Riemann sum</u> for f is

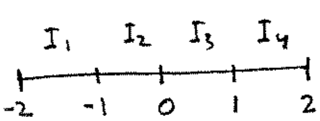
$$R_P = \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x_i$$

1a)

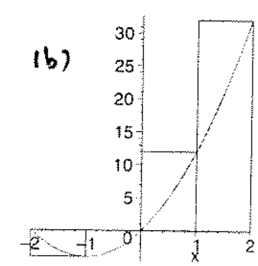


<u>Exercise 1</u> Let $f(x) = 4x^2+8x$
$-2 \le x \le 2$

Partition the interval into 4 equal-length subintervals of $\Delta x_i = 1$, i.e.



1b)

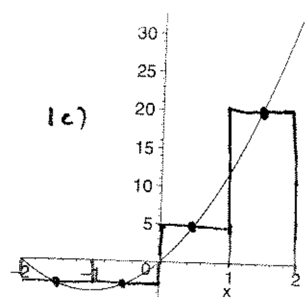


Calculate R_P using

1a) left endpoints

1b) right endpoints

1c)



1c) midpoints

If we used a finer partition, say with $n=100$ points and right endpoints, we'd get:

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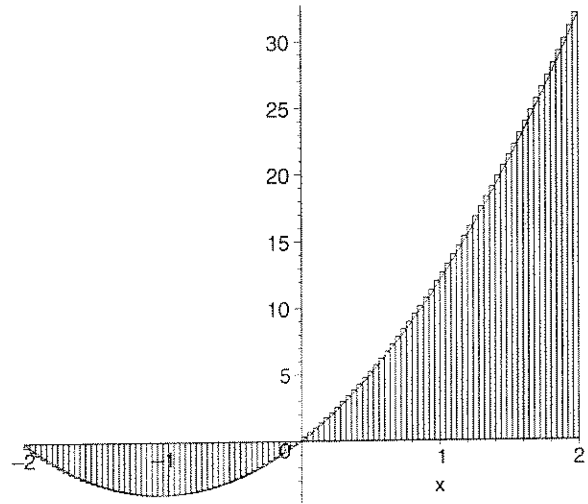
> rightbox(4*x^2+8*x,x=-2..2,100);
```




```
> R[P]:=rightsum(4*x^2+8*x,x=-2..2,100);
```

$$R_P := \frac{1}{25}\left(\sum_{i=1}^{100}\left(4\left(-2+\frac{i}{25}\right)^2 - 16 + \frac{8\,i}{25}\right)\right)$$

```
> evalf(%);
```

21.97760000

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>
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<u>Definition A</u> The <u>norm</u> of a partition P, written $\|P\|$, is the maximum of the widths Δx_i

<u>Exercise 2</u> : What is $\|P\|$ in Exercise 1?

What is $\|P\|$ at the top of this page?

<u>Definition B</u> If $\lim_{\|P\|\to 0} R_P$ exists then $f(x)$ is <u>Riemann Integrable</u> on $[a,b]$,

and we define "the (definite) integral from $x=a$ to $x=b$, of $f(x)$ w.r.t. x"

$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} R_P .$$

<u>Theorem</u> : If f is continuous on $[a,b]$, then the $\int_a^b f(x)\,dx$ exists.

Its value can be interpreted as the sum of the signed areas bounded between $y=f(x)$ and the x-axis, positive signs for regions above the x-axis, negative signs for regions below.

Exercise 3 Using $\sum_{i=1}^{n} i = \frac{1}{2}(n)(n+1) = \frac{1}{2}(n^2+n)$

$$\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(2n^3+3n^2+n)$$

Compute $\displaystyle\int_{-2}^{2} 4x^2 + 8x \, dx$

Use P_n, a subdivision of $[-2,2]$ into n equal-length subintervals, and right endpoints for $\bar{x}_i$.

$\Delta x =$

$I_i =$

$R_P =$

general subdivision of $[a,b]$ into n equal-length subintervals

$\Delta x = \frac{b-a}{n}$

$I_1 = [x_0, x_1] = [a, a+\Delta x]$

$I_2 = [x_1, x_2] = [a+\Delta x, a+2\Delta x]$

$I_3 = [x_2, x_3] = [a+2\Delta x, a+3\Delta x]$

$\vdots$

$I_i = [x_{i-1}, x_i] = [a+(i-1)\Delta x, a+i(\Delta x)]$

$\vdots$

$I_n = [x_{n-1}, x_n] = [a+(n-1)\Delta x, b]$.

Exercise 4 The Fundamental Theorem of Calculus asserts that

$$\int_a^b f(x)\,dx$$

actually equals $F(b)-F(a) := F(x)\Big]_a^b$, where F is any antideriv of f.

Use FTC to check your answer to # 3.