Math 1210-2
Monday Nov. 5

pages 2-3 Friday, on differential equations (Since we mostly just graphed on Friday.)

Why differentials and antidifferentiation work to solve separable DE's:

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

mathematically "correct"

$$g(y)\frac{dy}{dx} = f(x)$$

If $D_y G(y) = g(y)$
and $D_x F(x) = f(x)$, then by chainrule:

$$D_x G(y) = D_x F(x)$$

so $G(y) = F(x) + C.$

gives y implicitly as a function of x

method of separating variables

$$g(y)\,dy = f(x)\,dx$$

$$\int g(y)\,dy = \int f(x)\,dx$$

$$G(y) + C_1 = F(x) + C_2$$

$$G(y) = F(x) + C$$

same!

<u>Exercise 1</u> (Example 5 p.207).

Earth's gravitational attraction F on an object of mass m at a distance r from the center of the earth is

$$F = -mg\frac{R^2}{r^2}$$ (attraction is towards earth center.)

where R is the radius of the earth ≈ 3960 miles
$g \doteq -32$ ft/sec^2, as usual

Neglecting friction, what is "v_0" - the smallest initial velocity (called "escape velocity") an object can be given so that it does not fall back to earth?

Newton: $m\frac{dv}{dt} = -mg\frac{R^2}{r^2}$

$$v = \frac{dr}{dt}$$

trick: $\frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \frac{dv}{dr}v$

so $mv\frac{dv}{dr} = -mg\frac{R^2}{r^2}$

Solve! $\begin{cases} v\frac{dv}{dr} = -g\frac{R^2}{r^2} \\ v(R) = v_0 \end{cases}$

$\sqrt{2gR} \approx 6.93$ miles/sec.