Math 1210-2
November 30, Friday

Curve lengths.
& surface areas for surfaces of revolution

We derived the formula for length of parametric curves:

$$x = f(t) \quad a \le t \le b$$
$$y = g(t)$$

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2}\, dt$$

Let's check! (This continues page 5 Wednesday)
Find the following lengths: (Sketch curve when appropriate.)

Exercise 1 $\quad x = a\cos t \quad 0 \le t \le 2\pi$
$\quad y = a\sin t$

Exercise 2 (See Wed. pg 3-4)

$x = 2t+1$
$y = t^2 - 1 \quad 0 \le t \le 3$

Exercise 3 $\quad x = t^2 \quad 0 \le t \le 1$
$\quad y = 3t^2 + 4$

Exercise 4 $y = 3x + 4, \quad 0 \le x \le 1$ (use x as the parameter "t")

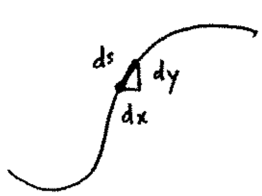
(compare to #3)

Exercise 4b $y = f(x), \quad a \le x \le b$

Exercise 5 $x = \frac{1}{3}y - \frac{4}{3}, \quad 4 \le y \le 7$ (use y as the parameter "t")

Exercise 5b $x = g(y), \quad c \le y \le d$

Summary

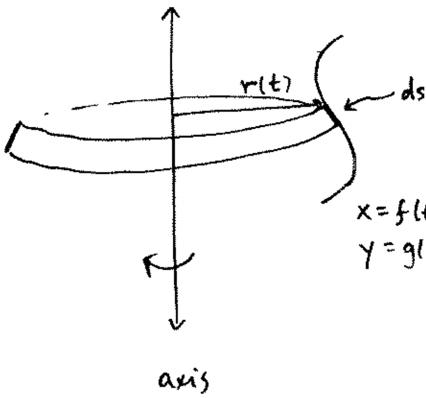


$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \qquad x = f(t),\ y = g(t) \quad a \le t \le b$$

$$L = \int ds = \int \sqrt{dx^2 + dy^2} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad y = f(x), \quad a \le x \le b$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy \qquad x = g(y), \quad a \le y \le b$$

Surface area of revolution :



$x = f(t)$
$y = g(t) \quad a \le t \le b$

Rotate arc of length ds,
get a ribbon of area

$$\Delta A \approx (2\pi \bar{r})(ds) \qquad \bar{r} = \text{average radius}$$

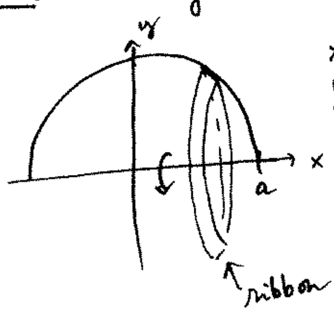
formula exact if arc is a
line segment, and ribbon
is a piece of a cone surface

(see #31. p 301).

Leads to surface area formula, if you rotate the curve about an axis :

$$A = \int_a^b \underbrace{2\pi \overset{\substack{\text{radius, i.e. distance from } (f(t), g(t)) \\ \text{to the axis} \\ \downarrow}}{r(t)}}_{\text{circum}} \underbrace{\sqrt{f'(t)^2 + g'(t)^2}\, dt}_{ds}$$

Exercise 6 : Verify the formula for the surface area of a sphere



$x = a\cos t \qquad 0 \le t \le \pi$
$y = a\sin t$