

Math 1210-2

Wednesday Nov. 21

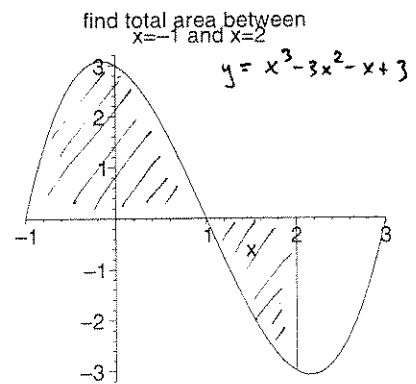
5.1 : Areas of plane regions

We motivated our discussion of Riemman sums and $\int_a^b f(x) dx$ by thinking about "signed area". Now that we're comfortable using FTC to compute definite integrals we can find the area of more interesting regions

Exercise 1 : Find to total area (not the signed area) between the graph of

$$y = x^3 - 3x^2 - x + 3 = (x-3)(x^2-1) = (x-3)(x+1)(x-1)$$

and the x -axis, between
 $-1 \leq x \leq 2$

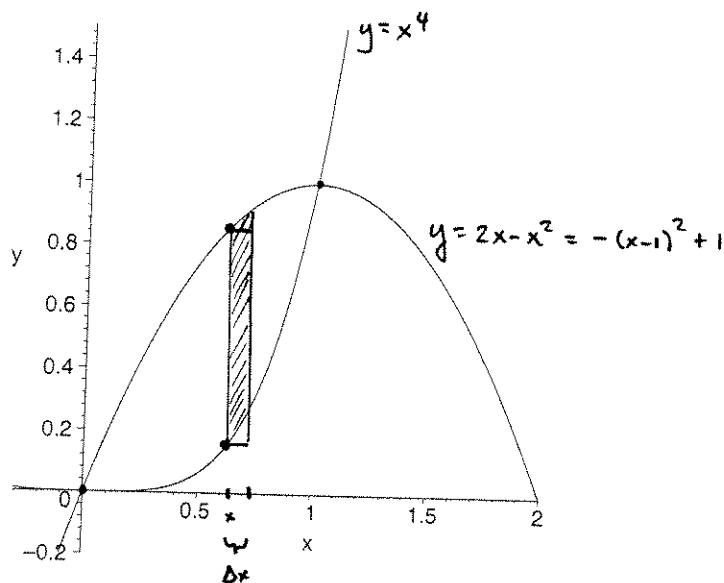


Exercise 2 Find the area of the region between $y=x^4$ and $y=2x-x^2$.

First, find an expression which approximates the area

$$\Delta A$$

of the almost rectangular region between the two graphs, with vertical sides at x and $x+\Delta x$. Visualize how a corresponding Riemann sum would look, and write down the definite integral you'd get in the limit. Finally, compute the area.

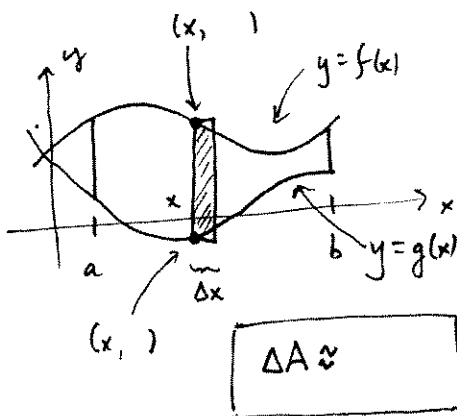


$$\Delta A \approx$$

Exercise 3 Suppose $g(x) \leq f(x)$ on the interval $[a, b]$.

Derive a formula for the area between the curves $y=f(x)$, $y=g(x)$, $a \leq x \leq b$ for

Area =



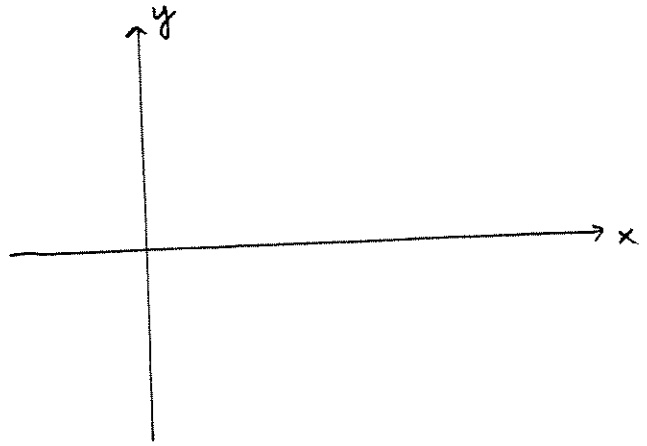
$$\Delta A \approx$$

Sometimes it works better to slice horizontally rather than vertically!

Exercise 4 Find the area between the curves $y = x - 1$
 $x = 3 - y^2$.

Sketch first!

4a) Use vertical slicing. You will need two definite integrals!



4b) Use horizontal slicing. It will be easier!

Exercise 5: Suppose $h(y) \leq k(y)$ for $c \leq y \leq d$. Find a formula for the area between the curves $x = h(y)$ and $x = k(y)$, for $c \leq y \leq d$. Use horizontal slicing

