Math 1210-2
Friday Nov. 2

§3.8-3.9 : Antidifferentiation &
Intro to differential equations

Exercise 1 Finish $\int 7\cos 5x + 3\sin^2(2x)\cos 2x \, dx$

Exercise 2 Can you find $\int x\cos x \, dx$? (This one's tricky!)

13.9 Differential Equations

A <u>differential equation</u> is an equation involving an unknown function $y = f(x)$, its derivatives, and the variable, e.g. x. (Further conditions on y may also be specified) e.g.

<u>Solving the differential equation</u> means finding the solution(s) $y = f(x)$.

<u>Examples:</u>

<u>Exercise 3</u>: Solve $\begin{cases} \frac{dy}{dx} = x & \leftarrow \text{differential equation} \\ y(0) = 1 & \leftarrow \text{initial condition} \end{cases}$

<u>Exercise 4</u> The acceleration of an object moving along a coordinate line is given by $a(t) = (2t+3)^{-3}$ meters/sec^2. If the velocity at $t = 0$ is 36 m/s, find the velocity 2 seconds later.

Exercises 3 & 4 are relatively easy, because for the differential equation

$$\frac{dy}{dx} = f(x)$$

You just solve by antidifferentiation,

$$y = \int f(x)\,dx$$

It turns out we can use the chain rule backwards (via differentials), to solve slightly harder differential equations of the form

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}.$$

These are called <u>separable</u> differential equations.

<u>Exercise 5</u> (Example 2 p. 205). Solve

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{x + 3x^2}{y^2} \\ y(0) = 6 \end{cases}$$