Prove FTC1, Monday notes, pages 2-4.

\[
\text{FTC1: } \frac{d}{dx} \int_a^x f(t) \, dt = f(x) \\
\text{FTC2: } \int_a^b f(x) \, dx = F(b) - F(a) \quad \text{if } F \text{ is any antiderivative of } f.
\]

Exercise 1: Combine FTC1 and FTC2 to compute the following derivatives, without first actually calculating the definite integral. When possible, check your answer by calculating the definite integral, then differentiating.

1a) \( D_x \int_0^x \sin t \, dt \)

1b) \( D_x \int_{2x}^{3x} t^2 \, dt \)

1c) \( D_x \int_1^{\cos x} \frac{1}{t} \, dt \)
Note: for \( a < b \), FTC2 says \( \int_a^b f(t) \, dt = F(b) - F(a) \).

In order to be consistent with this fact, we define

\[
\int_a^a f(t) \, dt = 0, \quad \int_b^a f(t) \, dt = -\int_a^b f(t) \, dt
\]

(we secretly used this fact in Exercise 1) \( = F(a) - F(b), \) i.e. \( F(x) \bigg|_a^b \).

**Exercise 2** 
Find

\[
\int_2^0 \frac{t}{\sqrt{3t^2 + 1}} \, dt
\]

---

**Substitution in Definite Integrals**

Recall (?)

\[
\int f(g(x)) \, g'(x) \, dx = F(g(x)) + C
\]

which we work out in actual problems using \( u = g(x) \) 
\[ du = g'(x) \, dx \]

Thus also

\[
\int_a^b f(g(x)) \, g'(x) \, dx = F(g(b)) - F(g(a)) = \int_{g(a)}^{g(b)} f(u) \, du
\]

**Exercise 3** 
Rework Exercise 2, by converting the \( x \)-limits into \( u \)-limits.
# Review Sheet for Exam 3

**Problem Sessions:**

- Wed Nov 14 7-8:30 pm, WEB103
- Thurs Nov 15 9:40-10:30 am, TINTA 355 (here, the usual problem session)

Exam is this Friday, Nov 16.

## Chapter 3 (Except 3.7, Newton's Method) Chapters 3.1-4.4

### Max-min problems

3.1 Extreme values and critical points
3.2 Monotonicity & concavity
3.3 Local extrema, 1st & 2nd derivative tests
3.4 Practical max-min problems:
   - Express f(x) to be extremized in terms of 1 or more variables
   - Use constraint equations to find a function of just 1 variable which you wish to extremize on some interval domain
   - Use Calc. to find extremal value.

### Graphing with Calculus

3.5 (also 3.1-3.3),
   - Inc/Dec
   - CU/CD (& inflection points)
   - Intercepts
   - Asymptotes
   - Limits related to asymptotes
   - Sample points
   - Plot!

### Mean Value Theorem (for derivatives)

3.6: Statement of theorem; testing theorem for given $f(x)$ and interval $[a,b]$.

### Antiderivatives

3.8: sum rule, constant multiple rule, power rule; taylors series
   - Substitution using differentials (chain rule in reverse)
3.9: solving 1st order differential equations by antiderivation. Separable DE's.

## Chapter 4.1-4.4

### The definite integral and Riemann sums

4.1-4.2: Computing Riemann sums, summation notation
   - Taking the limit as $\Delta x \to 0$ in special cases,
     $(\Delta x = \frac{b-a}{n}, \ n \to \infty)$
   - Providing you're given "magic formulas" interpreting definite integrals as signed area

### Computing definite integrals (and their derivatives), using FTC2, FTC1:

4.3-4.4: Theorem statements. Integral properties
   - How to compute definite integrals via antidifferentiation
   - Substitution in definite integrals derivatives of integral expressions (FTC1 + chain rule)
   - Applications to area, a net change as an integral of rate of change

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<table>
<thead>
<tr>
<th>Quiz</th>
<th>Class</th>
<th>Webwork</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1</td>
<td>2</td>
<td>p211 8</td>
</tr>
<tr>
<td>8/3</td>
<td>2</td>
<td>3</td>
<td>9,43</td>
</tr>
<tr>
<td>10/26 #1</td>
<td>4</td>
<td>9-15</td>
<td>p211-2/12 40, 42, 43</td>
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