

Name SOLUTIONS

Student I.D. \_\_\_\_\_

Math 1210-2  
Exam #3b  
November 16, 2007

Please show all work for full credit. This exam is closed book, closed note, closed calculator. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful to not spend too long on any one problem. Good Luck!!

Score	POSSIBLE
1 _____	30
2 _____	25
3 _____	20
4 _____	25
TOTAL _____	100

- 1) Consider the function  $f(x) = x^4 - 4x^3$ .  
 1a) What are the intervals on which  $f(x)$  increasing or decreasing?

(5 points)

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$$

Sign  $f'$ :  $\frac{\text{----} \quad 0 \text{---} \quad 0 \text{+++}}{\text{DEC} \quad 0 \quad \text{DEC} \quad 3 \quad \text{INC}}$

(so, actually decreasing on  $(-\infty, 3]$  and increasing on  $[3, \infty)$ )

- 1b) What are the intervals on which  $f(x)$  is concave up or concave down?

(5 points)

$$f''(x) = 12x^2 - 24x$$

$$= 12x(x-2)$$

sign  $f''$   $\frac{\text{+++} \quad 0 \text{--} \quad 0 \text{+++}}{\text{CU} \quad 0 \quad \text{CD} \quad 2 \quad \text{CU}}$

- 1c) Find all local extreme values of  $f(x)$ , and where they occur. Identify whether each is a local maximum or a local minimum value. Which of these is a global extreme value?

(5 points)

critical pts (stationary)

@  $x = 0, 3$



$f(0)$  is not a local extreme value.

$f(3)$  is local and global minimum since  $f$  is decreasing for  $x \leq 3$  and increasing for  $x > 3$

- 1d) Find all  $x$  and  $y$ -intercepts for the graph of  $f$ .

(5 points)

$x=0 \Rightarrow y=0 \leftarrow y\text{-intercept } (0,0)$

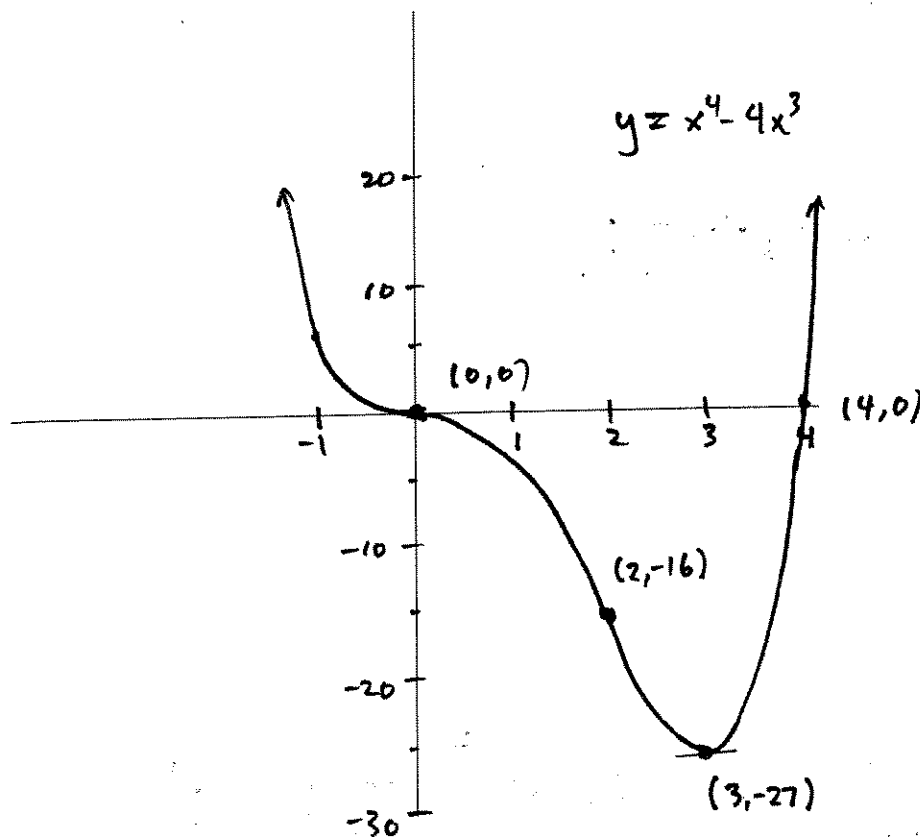
$y=0 \Rightarrow x^4 - 4x^3 = 0$

$x^3(x-4) = 0$

$x=0, x=4 \leftarrow x\text{-intercepts } (0,0), (4,0)$

1e) Create an accurate graph of  $y=f(x)$ , using all of your results from parts (1a)-(1d). Label all important points.

(10 points)



x	$x^4 - 4x^3$
0	0
3	-27
2	-16
4	0
1	-3
-1	5

$$3^4 - 4 \cdot 3^3 = 3^3(3-4) = -27$$

$$2^4 - 4 \cdot 2^3 = 2^4(1-2) = -16$$

$(0,0), (2,-16)$  are inflection points.

$(3,-27)$  is lowest pt. on graph (-27 is global minimum value)

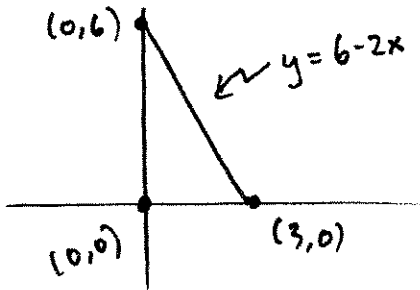
$(0,0)$  is a critical point but  $f(0)$  is not a local extreme value

$$(f'(0) = 0)$$

2) Consider the triangle in the first quadrant with edges along the x-axis, the y-axis, and the line  $y = 6 - 2x$ .

2a) Sketch the triangle and label the three vertices.

(4 points)



2b) Use geometry to compute the area of this triangle.

(4 points)

$$A = \frac{1}{2}bh = \frac{1}{2} \cdot 3 \cdot 6 = \boxed{9}$$

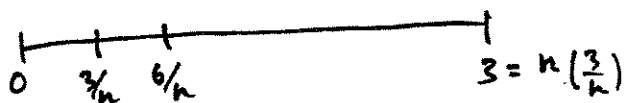
2c) Express the area of this triangle in terms of a definite integral, and compute the integral using the Fundamental Theorem of Calculus.

(5 points)

$$\int_0^3 (6 - 2x) dx = \left[ 6x - x^2 \right]_0^3 = 18 - 9 = \boxed{9}$$

2d) Now, in the last two parts of this problem, you will use the Riemann sum definition of definite integrals to recalculate the area a third time. First, divide the interval  $[0,3]$  into  $n$  equal-length subintervals. Decide how long each subinterval is, and use this information to figure out the endpoints on the  $i^{\text{th}}$  subinterval. Then, using the right-hand endpoints as your sample points, write down the Riemann sum for this function  $f(x) = 6 - 2x$ .

(7 points)



$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$I_i = [a + (i-1)\Delta x, a + i\Delta x]$$

$$= \left[ (i-1)\left(\frac{3}{n}\right), i\left(\frac{3}{n}\right) \right]$$

$$R_P = \sum_{i=1}^n f(\bar{x}_i) \Delta x_i = \sum_{i=1}^n \left[ 6 - 2\left(\frac{i3}{n}\right) \right] \left[ \frac{3}{n} \right]$$

2e) Use an appropriate magic formula on the blackboard to express the Riemann sum in 2d) in terms of  $n$ , and then take the limit as  $n$  approaches infinity. Hopefully you will recover the area you computed in the earlier parts of this problem!

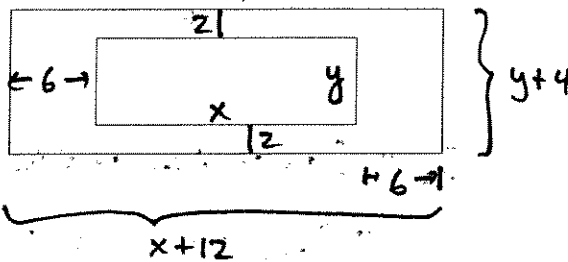
(5 points)

$$\begin{aligned} R_P &= \sum_{i=1}^n \frac{18}{n} - \frac{18i}{n^2} \\ &= \sum_{i=1}^n \frac{18}{n} - \frac{18}{n^2} \sum_{i=1}^n i \\ &= \left(\frac{18}{n}\right)(n) - \frac{18}{n^2} \cdot \frac{1}{2}n(n+1) \\ &= 18 - 9\left(\frac{n^2+n}{n^2}\right) \\ &= 18 - 9\left(1 + \frac{1}{n}\right) \end{aligned}$$

$$\int_0^3 6-2x dx = \lim_{n \rightarrow \infty} \left( 18 - 9\left(1 + \frac{1}{n}\right) \right) = 18 - 9 = \boxed{9}$$

3) You are given the task of designing next year's Calculus Carnival poster, of which thousands will be produced. You decide that you will need 300 square inches for the text and picture area, with margins of 6 inches on the sides and two inches on the top and bottom of the poster. What is the minimum area  $A$  of paper which will be required for each rectangular poster?

(20 points)



Minimize

$$A = (x+12)(y+4)$$

subject to

$$xy = 300$$

$$y = \frac{300}{x}$$

$$A(x) = (x+12)\left(\frac{300}{x} + 4\right)$$

$$= 300 + \frac{3600}{x} + 4x + 48 \quad \text{domain } x > 0$$

$$A'(x) = -\frac{3600}{x^2} + 4$$

stat pt:  $\frac{3600}{x^2} = 4 \Leftrightarrow x^2 = \frac{3600}{4} = 900, x = 30$  (since  $x > 0$ )

(  $A''(x) = -\frac{3600(-2)}{x^3} > 0$  for all  $x > 0$ , so  $A(30)$  is global min. )



$$\begin{aligned} x &= 30 \\ y &= \frac{300}{30} = 10 \end{aligned}$$

so paper dimensions are  $\boxed{42 \text{ by } 14}$   
 and area is  $\boxed{42 \cdot 14 (=588) \text{ in}^2}$

$$\begin{array}{r} 42 \\ \times 14 \\ \hline 168 \\ 42 \\ \hline 588 \end{array}$$

4) Compute the following:

4a)  $\int t^2 + \frac{3}{t^3} - 4 dt = \frac{t^3}{3} + 3 \frac{t^{-2}}{-2} - 4t + C$

$\underbrace{\hspace{10em}}_{3t^{-3}}$

(6 points)

4b)  $\int_1^8 \frac{10}{\sqrt{3x+1}} dx$

$u = 3x+1$   
 $du = 3 dx$

$= \frac{10}{3} \int_1^8 \frac{3 dx}{\sqrt{3x+1}} = \frac{10}{3} \int_4^{25} \frac{du}{\sqrt{u}} = \frac{10}{3} \int_4^{25} u^{-1/2} du = \frac{10}{3} \left[ 2u^{1/2} \right]_4^{25}$

$= \frac{10}{3} [10 - 4] = \frac{60}{3} = \boxed{20}$

(8 points)

4c)  $D_x \int_0^{x^3} \cos(t) \sin(t) dt = D_x [F(x^3) - F(0)] = f(x^3) \cdot 3x^2$

(6 points)

$= \cos(x^3) \sin(x^3) \cdot 3x^2$

(or, you can do this the long way by first computing def integral, probably via  $u = \cos t$  or  $u = \sin t$  substitution, then taking  $D_x$  of your answer.)

4d) All asymptotes for the graph  $y = \frac{2x^2+3x}{x-1}$ .

(5 points)

vert asymptote  $x=1$

diag: 
$$\begin{array}{r} 2x+5 \\ x-1 \overline{) 2x^2+3x} \\ \underline{2x^2-2x} \phantom{0} \\ 5x \phantom{0} \\ \underline{5x-5} \\ 5 \end{array}$$

$2x+5 + \frac{5}{x-1}$

so diag asymptote is

$\boxed{y = 2x+5}$

