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Name Solutions

Student I.D.

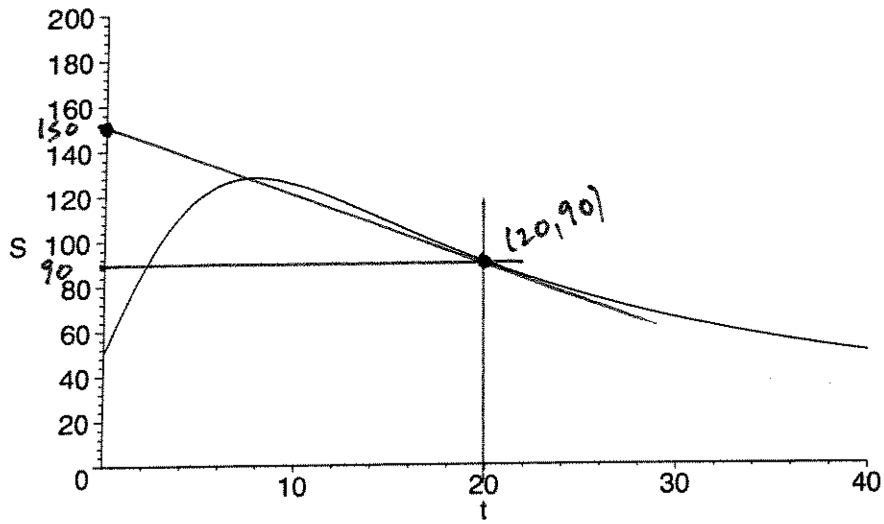
Math 1210-2
Exam #2b
October 19, 2007

Please show all work for full credit. This exam is closed book, closed note, closed calculator. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful to not spend too long on any one problem. Good Luck!!

Score

Score	POSSIBLE
1	25
2	25
3	25
4	25
TOTAL	100

1) Mathville is hit with another flu epidemic! The number S(t) of sick people at time t days after the epidemic begins is plotted below:



1a) Using the graph above, estimate how many people are sick on day t=20. (It might be helpful to fold your paper and use the edges to draw any lines you need.) Show work!

(5 points)

$S(20) \approx 90$ people ($\pm 10\%$ fine)

1b) Using the graph above, estimate how fast the number of sick people is decreasing on day t=20. Show work!

(8 points)

rate of change of = derivative of $S(t)$ = slope of graph $y = S(t)$

draw tang line at $t=20$, slope $\approx \frac{90-150}{20} = \frac{-60}{20} = -3$ people/day (so # is decreasing at 3 people/day)

1c) It turns out the sick people graph shown above is very close to the graph of the function

$$S(t) = \frac{20\,t + 50}{1 + 0.01\,t^2}.$$

Use this formula for S(t) to calculate values for how many people are sick on day t=20, and also to calculate how fast the number of sick people is decreasing on day 20. (You should get numbers close to the ones you found geometrically in parts (1a) and (1b) !)

(12 points)

$$S(20) = \frac{400+50}{1+.01(400)} = \frac{450}{5} = 90 \text{ people}$$

$$S'(t) = \frac{20(1+.01t^2) - (20t+50)(.02t)}{(1+.01t^2)^2}$$

$$S'(20) = \frac{20(1+4) - (400+50)(.4)}{(1+4)^2}$$

$$= \frac{100-180}{25} = \frac{-80}{25} = \frac{-16}{5} = -3.2 \text{ people/day}$$

$\rightarrow$ so dec. @ 3.2 people/day

2a) Use the limit definition of derivative to compute $D_x\left(\frac{1}{2x-3}\right)$.

(12 points)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{2(x+h)-3} - \frac{1}{2x-3}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{2x}-\cancel{3} - [2(\cancel{x}+h)-\cancel{3}]}{[2(x+h)-3][2x-3]}\right]$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}} \frac{-2\cancel{h}}{[2(x+h)-3][2x-3]} = \frac{-2}{[2x-3][2x-3]}$$

$$\boxed{= \frac{-2}{(2x-3)^2}}$$

2b) Use an appropriate differentiation rule to check your answer to part (2a).

(5 points)

$$f(x) = (2x-3)^{-1}$$

$$f'(x) = -(2x-3)^{-2}2 \quad \text{(chain rule).}$$

$$= \frac{-2}{(2x-3)^2}$$

or $f(x) = \frac{1}{2x-3}$

$$f'(x) = \frac{0-1\cdot 2}{(2x-3)^2} \quad \text{(quotient rule)}$$

$$= \frac{-2}{(2x-3)^2}$$

2c) Continuing to discuss the function $y = f(x) = \frac{1}{2x-3}$, use the fact that $f(2) = 1$ and differentials to approximate $f(2.05)$. ~~Compare your approximation to the exact value of f(2.05), which is slightly different.~~

(8 points)

$$y = f(x)$$

$$dy = f'(x)dx$$

$$f(x) = \frac{1}{2x-3}$$

$$f'(x) = \frac{-2}{(2x-3)^2}$$

$$f'(2) = -2$$

$$x = 2$$

$$y = f(2) = \frac{1}{4-3} = 1$$

$$dx = .05$$

$$dy = f'(2)\,dx$$

$$= -2(.05)$$

$$= -.1$$

$$f(x+\Delta x) = y + \Delta y$$

$$\approx y + dy$$

$$= 1 - .1$$

$$f(2.05) \approx \boxed{= .9}$$

3) Compute the following derivatives.

3a) $D_t y$ for $y = t^3 + \frac{8}{t^2} - 7.5$. $\quad 8t^{-2}$

(6 points)

$$D_t y = 3t^2 + 8(-2)t^{-3}$$
$$= 3t^2 - \frac{16}{t^3}$$

3b) $D_x y$ for $y = \underset{f}{(4x^3 - 7x)}\underset{g}{(2x+1)^2}$.

(6 points)

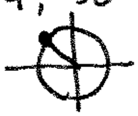
$$D_x y = f'g + fg'$$
$$= (12x^2 - 7)(2x+1)^2 + (4x^3 - 7x)\, 2(2x+1)^1\, 2$$

3c) $f'(\frac{\pi}{4})$ for $f(\theta) = [\sin(3\theta)]^2$. Evaluate all trig functions in your final answer.

(6 points)

$$f'(\theta) = 2[\sin(3\theta)]^1 \cos(3\theta)\, 3$$

$\theta = \pi/4,\ 3\theta = 3\pi/4$



$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

$$f'(\tfrac{\pi}{4}) = 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{2}} \cdot 3 = \boxed{-3}$$

3d) $D_x y$ at the point $(x,y)=(0,1)$, if y is given implicitly as a function of x by the equation

$$y^3 + y\cos(x) = 2x + 2.$$

(7 points)

(@ $\begin{matrix} x=0 \\ y=1 \end{matrix}$: $1 + 1\cdot 1 = 2\cdot 0 + 2$ ✓) so pt is on graph.

$$3y^2 y' + y'\cos x + y(-\sin x) = 2$$
$$y'(3y^2 + \cos x) = 2 + y\sin x$$

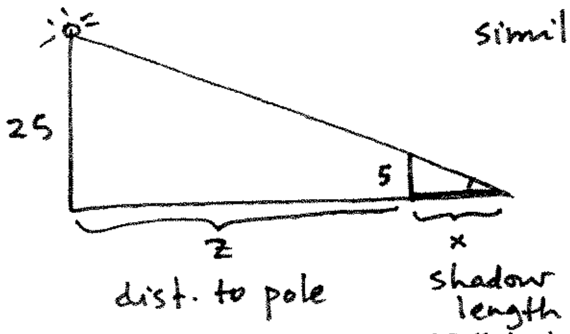
$$y' = \frac{2 + y\sin x}{3y^2 + \cos x}$$

@ $(x,y) = (0,1)$, $y' = \frac{2}{3+1} = \boxed{\frac{1}{2}}$

4) Sally is walking briskly away from a light pole 25 feet tall. The light at the top makes Sally have a shadow. At the instant when she is 50 feet from the pole Sally is walking away at a speed of 3 feet per second. Sally is 5 feet tall.

4a) How long is Sally's shadow at that instant? (5 points)

picture for 4a,b,c:



similar Δ's says

$$\boxed{\frac{x}{5} = \frac{x+z}{25}}$$

$$\Rightarrow 5x = x + z$$

$$\boxed{4x = z}$$

when $z = 50$

$$\boxed{x = \frac{z}{4} = 12.5 \text{ ft}}$$

4b) How fast is the length of Sally's shadow growing at that instant? (15 points)

@ $z = 50$ ft

$z'(t) = 3$ ft/sec

find $x'(t)$:

$$\boxed{4x = z}$$

$$\Rightarrow 4x'(t) = z'(t)$$

so $z'(t) = 3$

$$\Rightarrow \boxed{x'(t) = 3/4 \text{ ft/sec.}}$$

4c) How fast is the tip of Sally's shadow moving away from the pole at that instant? (5 points)

Find $D_t\,(x+z)$.

$$D_t\,(x+z) = x'(t) + z'(t)$$

$$= 3/4 + 3$$

$$\boxed{= 3.75 \text{ ft/sec}}$$