133

Name Solutions

Student I.D. ____________________

Math 1210-1
Exam #1b
September 12, 2007

Please show all work for full credit. This exam is closed book, closed note, closed calculator. There are 100 points possible, as indicated below and in the exam. Since you only have 50 minutes you should be careful to not spend too long on any one problem. Good Luck!!

Score	POSSIBLE
1 ____________	30
2 ____________	20
3 ____________	15
4 ____________	15
5 ____________	20
TOTAL ____________	100

1a) Find all x and y-intercepts, find at least 2 more helpful points on the graph, and then carefully sketch, the graph of $y = -\frac{x^2}{2} + 8$.

(8 points)

x	y
~~0~~	~~8~~
4	0
-4	0
0	8
-2	6
2	6

x-intercepts:

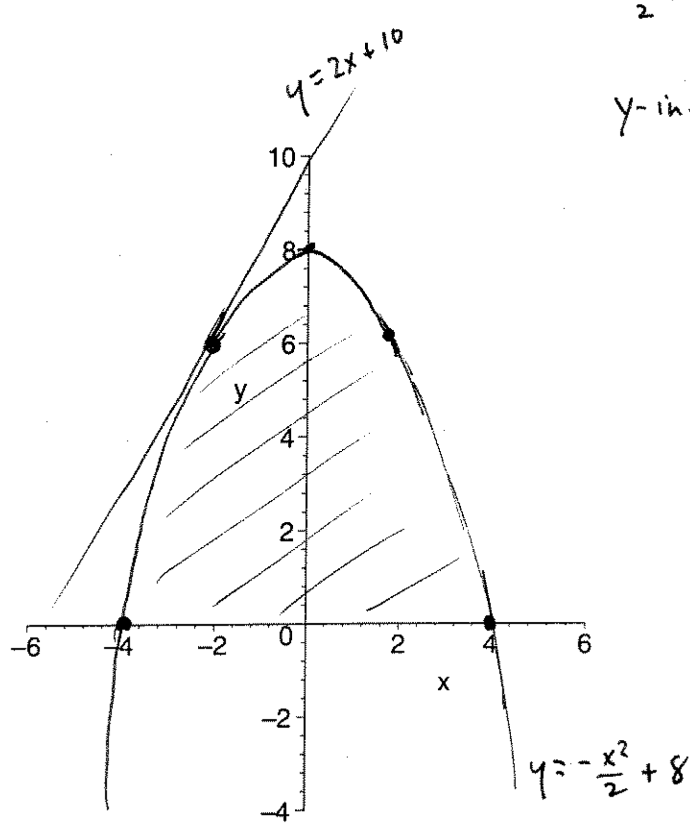
$0 = -\frac{x^2}{2} + 8$

$\frac{x^2}{2} = 8 \quad x^2 = 16$

$x = \pm 4$

y-intercepts $\quad y = 0 + 8$

$y = 8$



1b) Use the limit definition to compute the derivative of $f(x) = -\frac{x^2}{2} + 8$.

(10 points)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-\frac{1}{2}(x+h)^2 + 8 - \left[-\frac{1}{2}x^2 + 8\right]}{h}$$

$$= \lim_{h \to 0} \frac{-\frac{1}{2}(\cancel{x^2} + 2xh + h^2) \cancel{+8} + \cancel{\tfrac{1}{2}x^2} \cancel{-8}}{h}$$

$$= \lim_{h \to 0} \frac{-xh - h^2/2}{h} = \lim_{h \to 0} -x - \frac{h}{2} = -x$$

$\boxed{\text{So } f'(x) = -x}$

1c) Find the equation of the tangent line to the graph of $y = -\frac{x^2}{2} + 8$, at the point with x=-2. Add a sketch of this line to your picture on the previous page, making sure that it has the correct slope and y-intercept.

(5 points)

$f'(x) = -x$

$f'(-2) = 2 = \text{slope}$

$f(-2) = -\frac{4}{2} + 8 = 6$

point $(-2, 6)$

$y - 6 = 2(x+2)$

or

$y = 2x + 10$

1d) Shade the region bounded above by the parabola $y = -\frac{x^2}{2} + 8$, and below by the x-axis, on the previoius page's picture. Then find the area of this region.

(7 points)

$$A = \int_{-4}^{4} -\frac{x^2}{2} + 8 \, dx$$

$$= -\frac{x^3}{6} + 8x \Big]_{-4}^{4}$$

i.e. an antideriv. is $F(x) = -\frac{x^3}{6} + 8x$

and we want

$F(4) - F(-4)$

$$= \frac{-64}{6} + 32 - \left[+\frac{64}{6} - 32\right]$$

$$= -\frac{32}{3} + 32 - \frac{32}{3} + 32$$

$$= 32\left(2 - \frac{2}{3}\right)$$

$$= 64\left(1 - \frac{1}{3}\right)$$

$$= \frac{128}{3} \quad \text{or } 42\tfrac{2}{3}$$

or, since f is even, graph is symm w.r.t. y-axis,

and $A = 2\int_0^4 -\frac{x^2}{2} + 8 \, dx \ldots$

2) Shown below is the graph of y=|x|. Add graphs of the indicated equations to the picture, and explain how they are obtained from the original graph by reflection, scaling and translation. Make sure to label each graph.

2a) $y=|x+3|$. shift left 3

(5 points)

2b). $y=-|x|+2$ reflect across x-axis, then shift up by 2

(5 points)

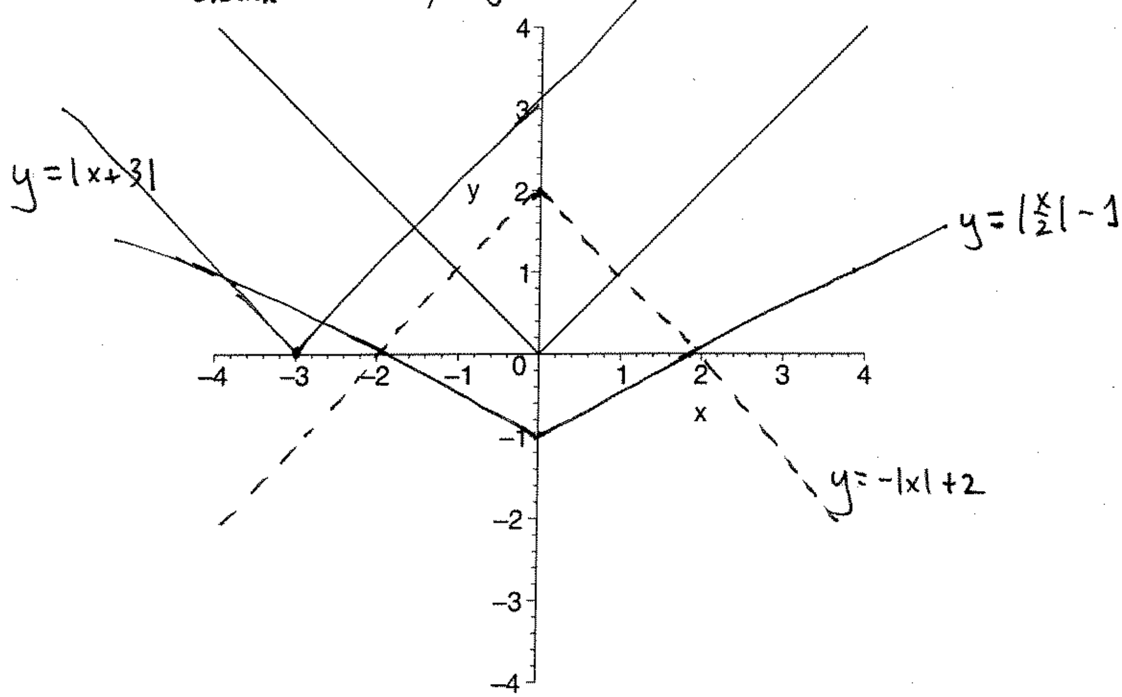
2c) $y+1=\left|\frac{x}{2}\right|$.

(5 points)

$y=|\frac{x}{2}|-1$ stretch horizontally by 2, shift down by 1

or

$y=\frac{1}{2}|x|-1$ ~~scale~~ shrink vertically by factor of ½, shift down by 1



2d) What is the interval of real numbers x which satisfies $|x+3|<0.05$?

(5 points)

$-.05 < x+3 < .05$

$-3-.05 < x < -3+.05$

$\boxed{-3.05 < x < -2.95}$

3a) Write down the Pythagorean identity and the angle addition identities for sine and cosine: (6 points)

$\cos^2 t + \sin^2 t = 1$

$\sin(\alpha+\beta) = \cos\alpha \sin\beta + \sin\alpha\cos\beta$

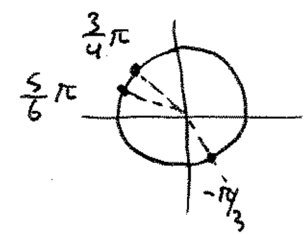
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

3b) Fill in the following values: (6 points)

$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$\sin\left(\frac{-\pi}{3}\right) = -\sqrt{3}/2$

$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$



$\frac{\pi}{3}$ rad $= 60°$

3c) Use the addition angle formula for sine, and the fact that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, to compute $\sin\left(\frac{\pi}{12}\right)$ (3 points)

$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} + \left(-\frac{\pi}{4}\right)\right)$

$= \cos\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) + \sin\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right)$

$= \frac{1}{2}\left(-\frac{1}{\sqrt{2}}\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$$\boxed{\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

4) Find the following limits if they exist, or say that they do not exist. Show your reasoning!

4a) $\lim_{x \to 0} \frac{|x|}{x}$ (5 points)

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

so $\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$

but $\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} -\frac{x}{x} = -1$

so $\lim_{x \to 0} \frac{|x|}{x}$ DNE (does not exist)

4b) $\lim_{h \to 10} \frac{h^2 - 8h}{\sqrt{h+6}}$ (5 points)

$= \frac{\lim_{h \to 10} h^2 - 8h}{\lim_{h \to 10} \sqrt{h+6}}$

$= \frac{\lim_{h \to 10} h^2 - 8 \lim_{h \to 10} h}{\sqrt{\lim_{h \to 10} h + 6}} = \frac{100 - 80}{\sqrt{16}} = \frac{20}{4} = \boxed{5}$

using our various limit thms.

4c) $\lim_{y \to 1} \frac{y^3 - 3y^2 + 2}{y^2 - 1}$ (5 points)

$\frac{0}{0}$. so factor.

$y^2 - 1 = (y-1)(y+1)$.

$$\begin{array}{r}
y^2 - 2y - 2 \\
y-1 \overline{)\, y^3 - 3y^2 + 2} \\
\underline{y^3 - y^2 } \\
-2y^2 + 2 \\
\underline{-2y^2 + 2y } \\
-2y + 2 \\
\underline{-2y + 2} \\
0
\end{array}$$

$\lim_{y \to 1} \frac{\cancel{(y-1)}(y^2 - 2y - 2)}{\cancel{(y-1)}(y+1)}$

$= \frac{1 - 2 - 2}{2} = \boxed{-3/2}$

5) On the moon's surface the vertical acceleration a due to gravity is given approximately by

$$a = -1.6 \ \frac{m}{s^2}$$

in the upwards direction. (Here we are using meters and seconds as our units of measurement, and the sign of the acceleration is negative because acceleration is in the downwards, i.e. negative, direction).

5a) If an object is projected upwards from the moon's surface with an initial velocity of 16 meters per second (and from an initial height s(0) = 0), find formulas for the velocity v(t) and height s(t), as functions of time t.

(15 points)

$a = -1.6$

$v = \int a(t)\,dt = -1.6t + C$

$v(0) = 16 = 0 + C$ so $C = 16$

$\boxed{v(t) = -1.6t + 16}$ m/s

$s = \int v(t)\,dt = \int -1.6t + 16\,dt$

$s = -.8t^2 + 16t + C$ $\quad s(0) = 0 = C$

$\boxed{s = -.8t^2 + 16t}$ m.

5b) What is the highest that the object in part (5a) gets?

(5 points)

when $v(t) = 0 = -1.6t + 16$

$t = 10$ s.

$s(10) = -.8(100) + 160$

$\boxed{= 80 \text{ m high}}$