Math 1210-2
Tuesday Dec 4

§5.5 Work

(Monday's notes have more physics than we have time for :( )

- Work done to move an object distance d against an opposing force F is

$$W = F\,d$$

when F is constant

force dist

example: To lift an object a height h on earth takes $\underset{\text{force}}{(mg)}\,\underset{\text{dist}}{h} = W$ work

- Thus, if $F(x)$, $a \le x \le b$ is a variable force

a ———— b

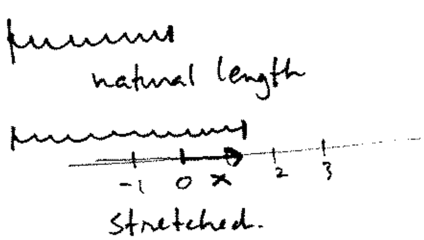
$$W \approx \sum_{i=1}^{n} F(\bar{x}_i)\Delta x_i \quad \text{is approximate work to move object from a to b}$$

$$W = \int_a^b F(x)\,dx \quad \underline{\text{is}} \text{ the work}$$

<u>Exercise 1</u> (p. 302). Hooke's Law (really just tangent line approx!), Says that to keep a spring stretched x units from its natural length you must exert a force proportional to x, i.e.

$$F(x) = kx.$$

(The spring's opposing force is $-kx$)



So, If a spring has natural length = .2 m and, 12 N force is required to stretch additional .03 m (not text #)

(a) Find the spring constant k (units?)

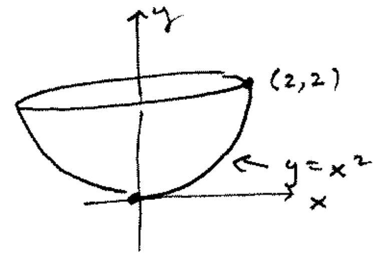
(b) How much work is required to stretch spring from .2 m to .3 m?

A different application of definite integrals to Work:

Exercise 2 (see p 303 for a related, but different example.)

A parabolic bowl, in the shape of a surface of revolution ($y = \frac{1}{2}x^2$, $0 \le x \le 2$ feet, about y-axis)

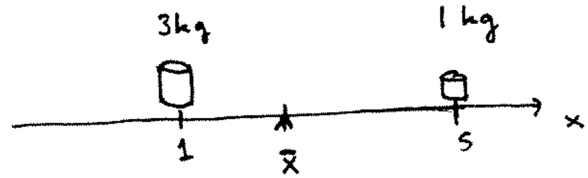
is full of water
(which weighs 62.4 lbs/ft^3)

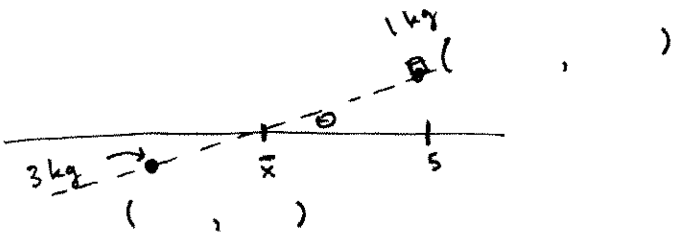


a) treating the bowl as weightless, how much work would it take to lift the entire bowl of water by **3** feet?
(so if the base is originally at $y = 0$, it will be lifted to $y = 3$).

b) How much work would it take to pump the water out of a faucet at height $y = 3$, from the original bowl-water position? (This should be less work than 2a), because not all the water is lifted 3 feet!)

<u>Exercise 3</u>: A mass of 3 kg is placed at $x = 1$ m
& a mass of 1 kg is placed at $x = 5$ m
on a weightless ruler.
Find the center of mass $\bar{x}$, defined to be the balance point where it takes no net work to "teeter totter" the masses on the ruler (working against gravity):





focus on the heights, and "mgh".

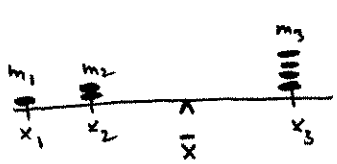
<u>Exercise 4</u>: Explain why if you have n masses, $m_1, m_2, \ldots m_n$ located at points $x_1, x_2, \ldots x_n$

then the balance point (called the center of mass) is

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

$\leftarrow$ called the moment M with respect to $x = 0$

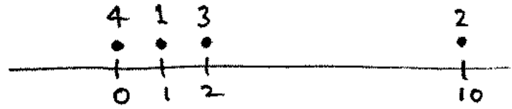
$\leftarrow$ called the total mass m



Exercise 5 What is the center of mass for masses of

$m_i = \ 4,\ 1,\ 3,\ 2$ kg at

at $x_i = \ 0,\ 1,\ 2,\ 10$ m ?



Tomorrow we calculusify
the notion of moments and centers of mass!

Explain this!!

webworks 10.3:

```
> with(plots):
Warning, the name changecoords has been redefined
> plot1:=plot([cos(theta)^3,sin(theta)^3,theta=0..2*Pi],color=black)
  :
  plot2:=plot([cos(theta)^2,sin(theta)^2,theta=0..2*Pi],color=black)
  :
  plot3:=plot([cos(theta),sin(theta),theta=0..2*Pi],color=black):
> display({plot1,plot2,plot3},title='circle,square edge,astroid');
```