Math 1210-2
Wed Aug 29

0.4: Graphs of equations, and how to translate, stretch, and reflect them

Definition A  The graph of an equation (with variables \( x, y \)) is the set of all points \((x, y)\) for which the equation is true. (A sketch of the graph is a picture which shows the graph)

Exercise 1  Fill in the table below and use it to sketch the graph of \( y = x^2 \), for \( |x| \leq 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>( \frac{3}{2} )</td>
<td>( \frac{9}{4} )</td>
</tr>
<tr>
<td>1.4</td>
<td>1.96</td>
</tr>
<tr>
<td>1.6</td>
<td>2.56</td>
</tr>
<tr>
<td>1.8</td>
<td>3.24</td>
</tr>
<tr>
<td>1.5</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Exercise 2  Find points on the graph of \( x^2 + y^2 = 1 \) and then use the Pythagorean Theorem to sketch the graph.
Exercise 3  Moving points in the plane

A point \((a, b)\) is plotted \((a<0, b<0, \text{in this case})\)
Numbers \(h, k\) are indicated \((h>0, k>0 \text{ in this case})\)
Plot the following points, describe geometrically.

(b) \((-a, b)\)

(ii) \((a, -b)\)

(iii) \((-a, -b)\)

(iv) \((b, a)\)

(v) \((a+h, b)\)

(vi) \((a, b+k)\)

(vii) \((a+h, b+k)\)

(viii) \((2a, 2b)\)

(ix) \((2a, 3b)\)

(x) \((\frac{1}{3}a, \frac{1}{3}b)\)
Reflecting graphs

A sketch of the graph \( F(x, y) = G(x, y) \)
(or, \( H(x, y) = 0 \))
is to be shown.

Exercise 4: Explain why the related equations have the particular reflected graphs.

Translating graphs

Exercise 5: Explain why the related equations have the particular translated graphs.

Notice: the truth seems "reversed!"

• If you replace \( x \) with \( x-h \)
  " " \( y \) with \( y-k \)
the graph moves \( h \) units to the right and \( k \) units up!!
Exercise 6: Explain why the related equations have the indicated graphs counterintuitive!

- If you replace $x$ with $cx$ and $y$ with $dy$
  - the graph is stretched by $\frac{1}{c}$ in $x$-dir
  - $\frac{1}{d}$ in $y$-dir!

Exercise 7: Sketch the graphs of (parts of)

(i) $y = -x^2$
(ii) $x = y^2$
(iii) $y = x^2 + 2$
(iv) $y = (x + 2)^2$
(v) $(x-3)^2 + (y+2)^2 = 1$
(vi) $(\frac{x}{3})^2 + (\frac{y}{2})^2 = 1$