Math 1210-2
Mon Aug 27

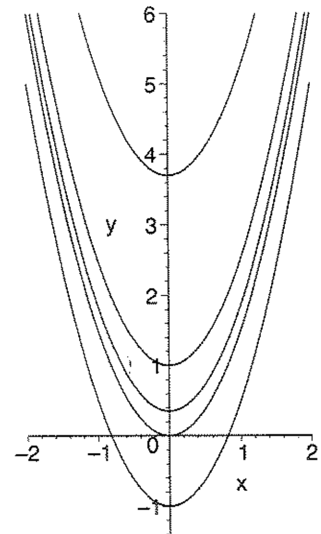
Quiz sol'ns are on our web page

§P.4 Antiderivatives of functions
(also called antidifferentiation)

<u>Definition A</u> If $f(x)$ is a function, then an <u>antiderivative</u> for f is a function having $f(x)$ as its derivative

(This is the reverse procedure to taking derivatives.)

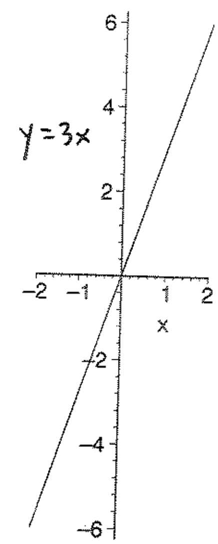
<u>Exercise 1</u>: Using your knowledge of derivatives and working backwards, find all the antiderivatives of $f(x) = 3x$ that you can find.
The pictures to the right are a hint.



graphs of possible antiderivatives

anti-differentiate

take derivative



<u>Definition B</u> If $f(x)$ is a function, the set of antiderivatives for $f(x)$ is denoted

$$\int f(x)\,dx$$

and is called the <u>indefinite integral of $f(x)$</u>.
(<u>with respect to x</u>)

Note, the "x" in "dx" says that x is your variable.

<u>Exercise 2</u>

$$\int 3x\,dx =$$

(You will rely on the theorem at the top of the next page to answer this exercise completely)

Theorem (We'll prove this in §3.8 of Vanberg, but it's believable.)

If $F(x)$ is an antiderivative of $f(x)$, then all other antiderivatives are of the form $F(x) + C$, where C is a constant.

i.e. $$\int f(x)\,dx = F(x) + C$$

Exercise 3 : Work backwards, as in Exercise 1, to find

3a) $\int x^3\,dx$

3b) $\int 4\,dx$

3c) $\int x^3 + 4\,dx$

3d) $\int 2x^5 + 3x^3 + x - 10\,dx$

3e) $\int -4t + 2\,dt$

3f)* $\int t^2x^3\,dt$

$\int t^2x^3\,dx$

Hopefully on page 2 we discovered

Theorem A If n is a non-negative integer, then

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \text{(where } C \text{ ranges over all constants)}$$

e.g.

$$\int 1\,dx =$$

$$\int x\,dx =$$

$$\int x^2\,dx =$$

$$\int x^3\,dx =$$

etc.

proof of Theorem A: Let $F(x) = \frac{x^{n+1}}{n+1}$. Then $F'(x) = \left(\frac{1}{n+1}\right)(n+1)\,x^n = x^n$.
This shows $F(x)$ is an antiderivative of x^n. You get every other antiderivative by adding (arbitrary) constants to $F(x)$. ∎

Theorem B If f and g are functions and "a" is a constant, then

(1) $\int a f(x)\,dx = a \int f(x)\,dx$

(2) $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

proof: If $F(x)$ is an antiderivative of $f(x)$, i.e. $F'(x) = f(x)$,
then $(aF)' = aF' = af$

so $aF(x)$ is an antiderivative of $af(x)$.

Thus
$$\int a f(x)\,dx = aF(x) + C_1 = a\,(F(x) + C) \qquad \left(C = \frac{C_1}{a}\right)$$
$$= a \int f(x)\,dx$$

Similarly, if $F' = f$ and $G' = g$, then
$$(F+G)' = f + g$$

So
$$\int f(x) + g(x)\,dx = F(x) + G(x) + C$$
$$= \int f(x)\,dx + \int g(x)\,dx \qquad ∎$$

Exercise 4 Find the antiderivative of x^2-1 which has value 2 when $x=-2$

Exercise 5 : The acceleration due to gravity on the earth's surface is $g = -32$ ft/sec² ($= -9.8$ m/sec²)

A ball is thrown straight upward with initial velocity of 128 ft/sec after which only gravity accelerates it. (We neglect friction.)

5a) What is the velocity $v(t)$, if $t=0$ sec. is when object is released?

5b) When does ~~object~~ ball reach maximum height?

5c) If ball was thrown from initial height of 6 feet, how high does it go?

5d)* When does it hit the ground?