Math 1210-2
Fri Aug 24 P.3 cont'd

Finish our discussion of how to take the derivative of any polynomial.
The key step is to figure out the derivative of

$$f(x) = x^k \qquad k \text{ a whole number}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^k - x^k}{h}$$

pattern:

Pascal's Δ records the coefficients

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & & & & & \end{array}$$

$$(x+h)^0 = 1$$
$$(x+h)^1 = 1x + 1h$$
$$(x+h)^2 = 1x^2 + 2xh + 1h^2$$
$$(x+h)^3 = \boxed{1}x^3 + \boxed{}x^2h + \boxed{}xh^2 + \boxed{1}h^3$$
$$(x+h)^4 = \boxed{}x^4 + \boxed{}x^3h + \boxed{}x^2h^2 + \boxed{}xh^3 + \boxed{}h^4$$
$$\vdots$$

notice the numbers in successive rows are obtained by adding the two nearest numbers ~~ab~~ in the row above. In fact the numbers are called the <u>binomial</u> <u>coefficients</u>, and there is a formula for them; the coefficient for $x^{k-j}h^j$ is $\binom{k}{j} = \frac{k!}{j!(k-j)!}$.

Deduce
$$(x+h)^k = x^k + khx^{k-1} + (\text{terms with at least a factor of } h^2)$$

So
$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^k - x^k}{h} = \frac{[\cancel{x^k} + khx^{k-1} + (\text{extra terms})] - \cancel{x^k}}{h}$$
$$\% = kx^{k-1} + (\text{terms with at least a factor of } h)$$

So
$$f'(x) = \lim_{h\to 0}(\%) = kx^{k-1}$$

Physics interpretation of derivative:

We defined

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

and interpreted it as the slope of a graph.

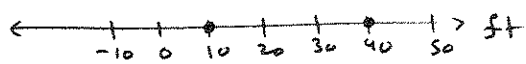
There are completely different interpretations of the same definition.

Suppose $s(t)$ is the location of an object a long a number line at time t. (The number line might be marked off in feet, and time might be measured in seconds.... or you might use miles and hours.)

Then $\dfrac{s(t+h) - s(t)}{h} = \dfrac{\text{displacement}}{\text{time}}$ $\quad \frac{ft}{sec}$ (or $\frac{miles}{hour}$)

is called average velocity

Exercise 1: If $s(0) = 10$, $s(2) = 40$ ($s(t)$ feet at time t sec)



What is the average velocity between $t = 0$ and $t = 2$ sec.?

So, it makes sense to define $\boxed{s'(t) = v(t) \text{ as the velocity}}$ at time t, since

$$\lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = s'(t)$$

is the limit of average velocities over shorter & shorter time intervals.

Exercise 2: Most of you know of a electro-mechanical device which computes derivatives very frequently. What is it?

Similarly $v'(t) = \lim_{h \to 0} \dfrac{v(t+h) - v(t)}{h}$ $\quad \left(\frac{ft/sec}{sec} \text{ or } ft/sec^2 \right)$

measures how fast velocity is changing and is called acceleration $a(t)$.

$$\boxed{v'(t) = a(t).}$$

<u>Exercise 3</u>: If a ball is dropped off the top of a 64 ft. high building, then its height s(t) seconds later is described by the formula

$$s(t) = -16t^2 + 64 \qquad \text{ft.}$$

3a) When does it hit the ground?

3b) What is its velocity when it hits?

3c) What is the acceleration function?