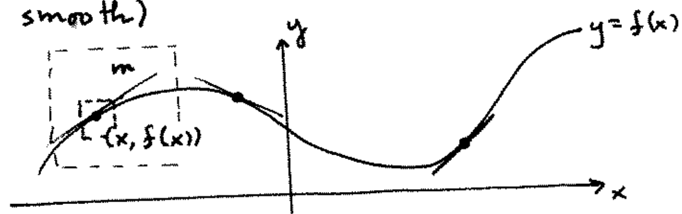
Math 1210-2
Tuesday Aug 21
JWB 335 9:40-10:30
do page 4 Monday!

course materials at
http://www.math.utah.edu/~korevaar/1210fall07

- add: Jianyu Wang's problem sessions Th 9:40-10:30 JWB 335 (here!)
- Webwork intro Wed (tomorrow)

P.2 Slope of a curve

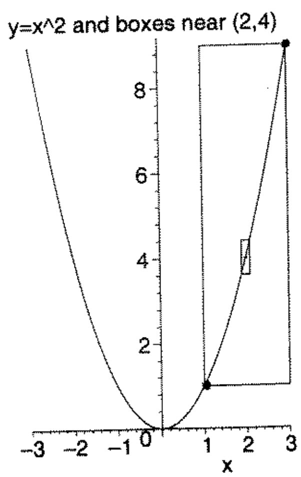
A graph $y=f(x)$ has a "slope" a each point $(x, f(x))$ on the graph (if the graph is smooth)

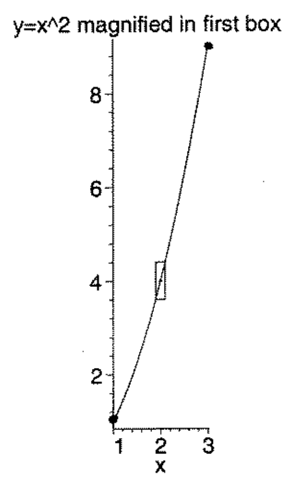


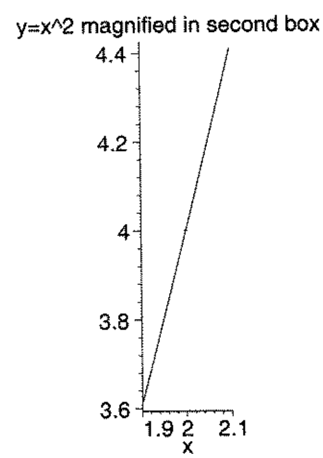
One way to visualize this slope is as the slope of the line "tangent" to the graph at $(x, f(x))$

Another way is to magnify the graph so much that it looks like a straight line (almost), and use that line's slope as an approximation.

Example 1 Estimate the slope of the parabola $y=x^2$ at the point $(2,4)$, by magnification







How can we calculate an exact value of the slope?

We are still focused on the graph $y=x^2$, and the point $(2,4)$

To save work we shall use $(2,4)$ as base point,

and pick nearby points (with x near 2) as second points with which to compute $\frac{\text{rise}}{\text{run}}$

Exercise 2

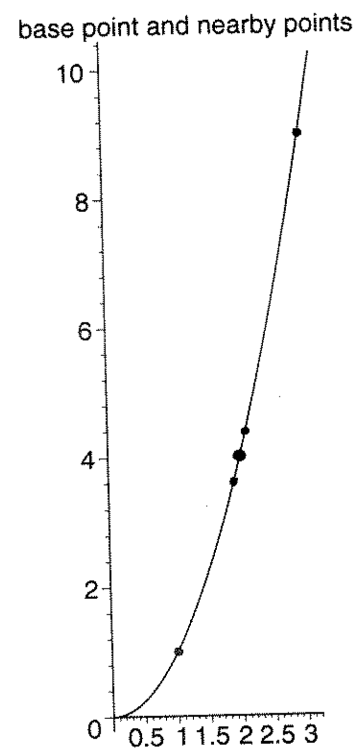
$(2,4)$ and $(3,9)$ $\quad \frac{\text{rise}}{\text{run}} = m_{sec} =$

$(2,4)$ and $(1,1)$ $\quad \frac{\text{rise}}{\text{run}} = m_{sec} =$

$(2,4)$ and $(2.1, (2.1)^2)$, $m_{sec} =$

$(2,4)$ and $(2.01, (2.01)^2)$, $m_{sec} =$

geometrically, we are computing the slopes of "secant lines" to the graph $y=x^2$, thru $(2,4)$: and other nearby points on the graph



Now for the exact value:

base point $(2,4)$

nearby points $(2+h, (2+h)^2)$

$$m_{sec} = \frac{(2+h)^2 - 4}{h} = \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h}$$

$$= 4 + h$$

as h approaches zero, m_{sec} approaches 4

We write

$$\lim_{h \to 0} 4 + h = 4$$

(and say, "the limit as h approaches zero of $4+h$ is 4")

$\boxed{\text{Exact slope at } x=2 \text{ on graph } y=x^2 \text{ is } 4}$

We can calculate the slope at any arbitrary point (x, x^2) on the graph $y = x^2$!

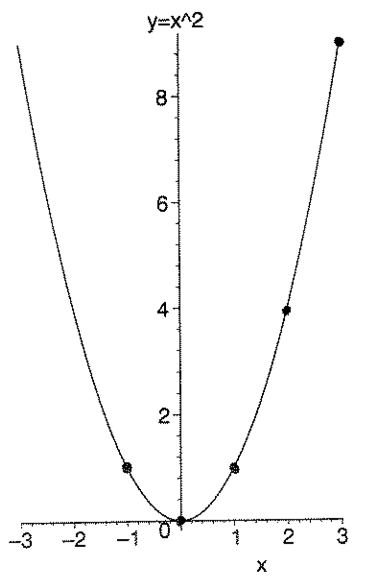
base point (x, x^2)

nearby point $(x+h, (x+h)^2)$

$$m_{sec} = \frac{\text{rise}}{\text{run}} = \frac{(x+h)^2 - x^2}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h} = \lim_{h \to 0} 2x + h = 2x$$

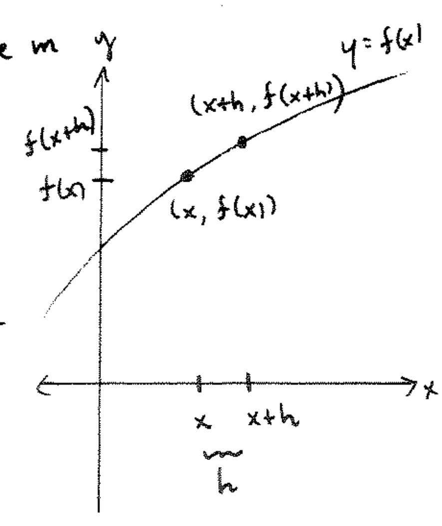
Exercise 3 What is the slope of $y = x^2$ at $x = 1$?
at $x = 0$?
at $x = -1$?
at $x = 3$?



Definition A: The graph of a function f is said to have slope m at a point $(x, f(x))$ on the graph provided

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = m$$

That is, provided the expression $\frac{f(x+h) - f(x)}{h}$ approaches m as h approaches zero. In this case we call the number m the derivative of f at x and denote it by $f'(x)$



<u>Exercise 4</u> Find the derivative of the constant function $f(x) = c$. (a constant)
Explain why your answer makes sense

<u>Exercise 5</u> Find the derivative of $f(x) = 5x - 7$.
Explain why your answer makes sense

<u>Exercise 6</u> Find $f'(x)$ for $f(x) = 2x^2 + 5x - 7$