

NAME: _____

Math 3210-2, Fall 1999

Test II

You have fifty minutes to work on these questions. You may use books and one page of notes, but not your neighbor's paper!

Please show all work. Erase or cross out unwanted work.

1. 20 POINTS. In several exercises, we used the following fact: If $A \subset \mathbf{R}$ is bounded above, and $s = \sup A$, then for all $\varepsilon > 0$, there exists $x \in A$ such that

$$s - \varepsilon < x < s.$$

Prove this fact from the definition of sup.

2. Fix a number a with $0 \leq a \leq 1$. Consider the sequence (s_n) defined recursively by

$$s_1 = a \text{ and } s_{n+1} = \sqrt{(s_n + 1)/2}$$

- (a) 20 POINTS. Use mathematical induction to show that $0 \leq s_n \leq 1$ for all n .

(b) 20 POINTS. Prove that (s_n) converges.

3. 20 POINTS. Recall that a series *converges absolutely* if $\sum_{k=1}^{\infty} |a_k|$ converges.

Suppose that $\sum_{k=1}^{\infty} a_k$ converges absolutely. Show that

$$\sum_{j=1}^{\infty} a_{k_j}$$

converges absolutely for any subsequence (a_{k_j}) of (a_k) .

4. 5 POINT EACH. Indicate whether each statement is true or false. Please write out the whole word “true” or “false”.

(a) $\sum_{k=1}^{\infty} \frac{2^k k!}{k^k}$ converges.

(b) Every sequence has a lim sup and a lim inf.

(c) If (s_n) is a bounded sequence and (s_{n_k}) a subsequence, then $\lim_{k \rightarrow \infty} s_{n_k}$ exists.

(d) If $\sum a_k$ converges and $\sum b_k$ diverges, then $\sum (a_k + b_k)$ diverges.