Course Information

Mathematics 3210-2: Foundations of Analysis — Fall 1999

INSTRUCTOR:	Dr. Holger Kley	LECTURES:	MWF 12:55–1:45 in JTB 110
OFFICE:	JWB 126	HOURS:	M 11:50–12:40, Tu 12:55–1:45, and by appointment.
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Overview

Math 3210–20 has two aims: the first, of course, is to teach you "the foundations of analysis", i.e., the rigorous mathematical theory underlying just about everything you have learned in one and multi-variable calculus. The second is to teach you how to do mathematics rigorously, i.e., how to do proofs. The two aims will be inextricably intertwined: you cannot learn any mathematics rigorously without being able to understand and do proofs, and it is almost impossible to learn how to do proofs without an extended example to practice on.

If you are someone who will never need to do more than apply some mathematical formulas, or perhaps use a computer algebra system to solve a differential equation, you do not need to take this course. On the other hand, if you plan on taking any higher level course in pure math—5310–20, etc.—or if you will ever need to answer a serious question asked by an ambitious high school student—you will need this course.

This is not an easy course. Many of you will find it frustrating, especially in the early weeks. You are expected to spend considerable time outside of class, time when you are alert and focussed, working on problems. To learn to do proofs, you must practice a lot. If you cannot do the typical homework problem—which inevitably consists of proving something or another—by the end of the semester, I cannot in good conscience, pass you.

If it sounds like I am trying to scare you, it's because I am; I want to scare you into treating this course seriously from the first day of class and allocating it the time it needs. On the other hand, I also want to encourage you. I firmly believe that most students at your level can learn this material. By the end of the year, one and multivariable calculus will no longer seem like a hodge-podge of formulas to be memorized, but rather a coherent theoretical entity *which makes sense!* At some point during the first semester, you will really understand the proof of some theorem. Hopefully, you will appreciate the innate beauty of it all. I promise to do my best to get you there.

Outline

Our goal for math 3210 is to cover most of Ross in the following order: §3, §5, §7–9, §4, §10–12, §1, §14, §15, §17–20, §23–34, and §37. This approach has us starting with sequences of real numbers: operations on sequences, convergence, Cauchy sequences, subsequences, series, etc. We then use convergent sequences to study functions: limits, continuity and *uniform continuity* (which will be a new idea for most of you).

It is at this point that we will prove three crucial theorems which are stated and used in first-year calculus, but (almost) never proved: the intermediate value theorem, the boundedness theorem (tot he effect that a continuous function on a closed interval is bounded) and the maximum value theorem (which state that a continuous function on a closed interval attains a maximum value).

We return to the study of sequences, but this time sequences of *functions.* Again we discuss convergence and the new notion of *uniform convergence.* Most of our examples in this section will be power series, and we will develop the theory of this subject quite a bit further than is done in first-year calculus.

Finally, we will discuss differentiation and integration, giving careful proofs of the chain rule, the inverse function theorem, and the fundamental theorems of calculus. As an application, we will give a rigorous definition of logarithm and exponential in terms of integrals, and use this definition to derive their familiar properties.

Details

Text: There are two required texts for the course: Ross [1] and Wade [4]. We will use Ross primarily in 3210 and Wade mostly in 3220, but there will be some overlap. In particular, Wade covers most of the material in Ross in a more terse style, and whenever possible, I will direct to parallel reading. I will supplement these texts with typed notes as needed. Also, in the references, I list some other helpful books which cover some of the course topics. Again, I will point to them whenever possible.

Lectures: My plan is to follow the general outline of Ross's exposition: please see 'outline' above. As the details of my presentation and point of view may differ from the book's, however, I strongly urge you to take notes, and to spend fifteen minutes reviewing your most recent notes before lectures. Also, please stop me to ask for clarification whenever the need arises. Don't be afraid to tell me that you can't read my handwriting; you won't be the first.

Homework: I will assign homework during most lectures, either from the book or problems of my own devising; homework will be due on Wednesdays. For every class after the due date, you will lose 33% of the points on your homework. While homework which is more than a week late will be corrected, you will receive no points for it. You will receive detailed comments on your homework; for this reason, it is important that you hand in even a partial solution to problems.

It is absolutely essential that you do the homework as we talk about the material. You will not truly understand the definitions and theorems presented in this course without applying them to problems, nor will you learn how to write mathematical proofs without quite a bit of practice. I have found that discussion can be an effective way to problem solving, so you are encouraged to work together on homework; I think you will find the process of explaining your ideas to others extremely helpful. A note of caution: it's probably best to try the problems by yourself first, so that you can bring something to the discussion.

Homework is also meant to teach the writing of mathematics. For that reason, I ask that after you have worked out a proof, each of you do your own final write-up. Papers which are exactly identical will make me suspicious...

Tests: There will be three in-class tests, one on Friday the 24th of September, one on Friday, October 22nd, and on Friday the 19th of November.

Finals: We will have a final exam at the regularly scheduled time: Thursday, December 16th, 11:30–1:30.

Grading: Course grades will be computed from homework grades (40%), test grades (15% each, drop the lowest one) and the final (30%). Letter grades will be assigned on a modified curve, meaning that there are no quotas for low grades: if you earn a B- because you score 75% on all of your work, you will receive a B-, even if the rest of the class has perfect scores. On the other hand, if you score 50% and the class average was 50%, you are guaranteed a B-.

Web page: I will maintain a home-page at `http://www.math.utah.edu/~kley/3110` for this course. On the page I will post any handouts that I give, tests and solutions, as well as all homework assigned. (Please note that in order to view the handouts, you will need to have Adobe Acrobat Reader installed on your machine. I will provide a link for down-loading this freeware.)

Mathematics Library: The mathematics library is located in JWB 121. The hours are M–F: 9–5 and SaSu: 11–4. Ongoing construction may affect these times; I will keep you advised.

ADA Statement: The Americans with Disabilities Act requires that reasonable accommodations be provided for students with physical, sensory, cognitive, systemic, learning and psychiatric disabilities. Please contact me at the beginning of the quarter to discuss any such accommodations for the course.

Miscellaneous: There will be no class on Monday, September 6th (Labor Day), Friday, October 8th (Fall Break), or Wednesday November 24th and Friday, November 26th (Thanksgiving Holiday.)

References

[1] Kenneth A. Ross, *Elementary analysis: The theory of calculus*, Undergraduate Texts in Mathematics, Springer, 1980.

[2] Douglas Smith, Maurice Eggen, and Richard St. Andre, *A transition to advanced mathematics*, fourth ed., Brooks/Cole, 1997.

[3] Michael Spivak, *Calculus*, third ed., Publish or Perish, Inc., 1994.

[4] William R. Wade, *An introduction to analysis*, Prentice Hall, 1995.