

# Math 2250-1 Maple Project 1

## Summer 2009

**Directions:** Hand in a single Maple document, printed and stapled neatly, which contains the answers to the exercises below. At the top of this document, you should create a text field with an appropriate title, the date, your name, and UID. Below this header, please answer the exercises in order. If an exercise calls for computations by hand, you can type up your solutions in a text field, or leave enough space so that you can hand-write your computation/explanation after printing the document.

**Maple Help:** Maple has an extensive Help menu! You can search for commands to find syntax, related commands, and extensive examples. Additionally, there are some introduction to Maple guides posted on our class webpage ([www.math.utah.edu/kitchen/2250Sum09](http://www.math.utah.edu/kitchen/2250Sum09)). You can also get help from tutors in the tutoring center, and follow the guidelines in the book as necessary.

**Exercises, Part 1:** These first four exercises are for warm-up, to get you used to Maple commands and syntax. All solutions should be performed in Maple.

1. **(Solving)** Solve the quadratic equation  $ax^2 + bx + c = 0$  and display its factorization for:
  - (a)  $a=8, b=22, c=15$
  - (b)  $a=1, b=2, c=6$
  - (c)  $a=2, b=16, c=32$
2. **(Functions and Plotting)** Define the functions below and plot them on the given domains:
  - (a)  $f(x) = \sin(10\pi x), 0 \leq x \leq 1$
  - (b)  $g(x) = |5 \ln(4 + x) - 1|, -1 \leq x \leq 3$
  - (c)  $h(t) = 7 + 6 \cos(\frac{\pi}{12}(t - 15)), 0 \leq t \leq 48$

In your solutions, show only the maple code and graphics. Please shrink the graphs to fit in about 1/4 page before printing.

3. **(Derivatives)** Compute the indicated derivatives:

- (a)  $\frac{dy}{dx}, y = \frac{(x-1)^2}{x^3+1} + xe^{-2x} + \cos(\ln(x^2 + 1))$
- (b)  $\partial_x z, \partial_y z, z = \cosh x \sinh 2y + e^y \tan 3x$
- (c)  $\partial_x \partial_y w, w = e^{\sin(xy) \tan(\ln(x^2 y^2 + 1))}$

4. **(Jacobian)** Compute the Jacobian matrix  $\begin{pmatrix} \partial_x z_1 & \partial_y z_1 \\ \partial_x z_2 & \partial_y z_2 \end{pmatrix}$ , and its determinant  $\begin{vmatrix} \partial_x z_1 & \partial_y z_1 \\ \partial_x z_2 & \partial_y z_2 \end{vmatrix}$  at  $x = y = 0$ . Do it in Maple and check your answer by hand.

$$z_1 = \sinh(x + 2y) + \ln(e^{xy} + 1) + \tan(x) \tan(y), \quad z_2 = \sec(\ln(x^2 y^2 + 1))$$

**Exercises, Part 2:** This part of your assignment is from section 1.5, First Order Linear Differential Equations, specifically from 1.5 Application: Indoor Temperature Oscillations, pages 56-58. This project is adapted Professor Gustafson's assignment, which he wrote for his Math 2250 section last

year. I recommend you work on this project with your text open. You may also want to consult the Maple hints on Professor Gustafson's page: <http://www.math.utah.edu/gustafso/s2009/2250mapleS2009.html>. (Even though our questions may not look exactly the same as his, they are related.)

Also, to see commands related to these specific problems, as well as plots of the solutions, see the file `MP1.mw` on the class webpage.

Here's the project set-up as explained in the text, page 56: You have no air conditioner or evaporative cooler for your home, it's summer, you live in the authors' home town of Athens Georgia, and the outside temperatures are oscillating periodically, with a period of 24 hours. In other words, we will assume the ambient temperature outside is given by

$$A(t) = a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t), \quad (1)$$

where the angular frequency  $\omega$  is  $2\pi$  radians per 24 hours, i.e.

$$\omega = \frac{1}{12}\pi$$

radians per hour. In the book example the 24-hour average temperature is  $80^\circ$  F, with a minimum of  $70^\circ$  at 4 a.m., and a maximum of  $90^\circ$  degrees at 4 p.m. This is consistent with the fact that the hottest part of the day usually occurs in the afternoon, and the coolest part is in the early morning hours. Using this information, we may write

$$A(t) = 80 - 10 \cos(\omega(t - 4))$$

and expand using the cosine addition angle formula, which you can find in any Calculus textbook:

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta), \quad (2)$$

This yields

$$A(t) = 80 - 5 \cos(\omega t) - 5\sqrt{3} \sin(\omega t). \quad (3)$$

(Check!! but don't hand in this verification.)

We wish to model the temperature inside the house, assuming no sources or sinks for heat exist inside, and using Newton's Law of Cooling. This model yields the linear DE

$$\frac{du}{dt} = k(A(t) - u)$$

for the indoor temperature  $u(t)$ , i.e.

$$\frac{du}{dt} + ku = k(a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t)). \quad (4)$$

The proportionality constant  $k$  reflects how quickly heat from the outside affects temperature change inside - the smaller  $k$  is, the more slowly the inside temperature changes for fixed difference  $A(t) - u$ , i.e. small  $k$  means good insulation.

1. Using the integration factor algorithm we've learned for solving first order linear DEs, and working by hand, show that the general solution to the DE (4) above is given just as the book claims in equation 4, page 57. In other words, that the solutions are

$$u(t) = a_0 + c_0 e^{-kt} + c_1 \cos(\omega t) + c_2 \sin(\omega t). \quad (5)$$

Here the constant  $c_0$  arises as a constant of integration, and is related to the initial temperature  $u_0$ . The constants  $c_1$  and  $c_2$  are determined by the insulation constant  $k$  and the angular frequency  $\omega$ , as shown on page 57. The formulas you derive for  $c_0$ ,  $c_1$ ,  $c_2$  agree with the book's display after equation (5). You may wish to use the book's integral tables, specifically #49, 50 on the back cover of our text.

2. Redo the computation for Exercise 1 using Maple to compute all the integrals you need. Your work here should all be done in a Maple worksheet. The Maple commands and output should be shown, along with any text explanation or command comments you find necessary.

Notice that the exponential part of the solution to (5) decays to zero (exponentially) as  $t \rightarrow \infty$ , and that what remains in the limit is also a solution (i.e. when the integration constant  $c_0 = 0$ ). We call this the steady periodic (sp) solution, and write

$$u_{sp}(t) = a_0 + c_1 \cos(\omega t) + c_2 \sin(\omega t). \quad (6)$$

3. Use Maple to crunch the numbers and show that with the Athens Georgia temperature data (i.e. the ambient temperature  $A(t)$  given by equation (1), and insulation constant  $k = 0.2$  we get

$$u(t) = 80 + (e^{-0.2t})(u_0 - 82.3351) + 2.3351 \cos\left(\frac{\pi t}{12}\right) - 5.6036 \sin\left(\frac{\pi t}{12}\right) \quad (7)$$

and so also,

$$u_{sp}(t) = 80 + 2.3351 \cos\left(\frac{\pi t}{12}\right) - 5.6036 \sin\left(\frac{\pi t}{12}\right) \quad (8)$$