Name KEY

Student ID # ____________

Math 2250
Summer 2009
S. Kitchen

EXAM I
Friday, June 26, 2009

Problem	Points	Score
1.	15	
2.	20	
3.	15	
4.	15	
5.	15	
6.	20	
	TOTAL	

(15 points) 1. Solve the first order linear differential equation $y' + \cos(x)y = \cos(x)$.

Hint: For *one* of the integrals you will need to compute, $u = \sin x$ will be a useful substitution!

Integrating factor: $\rho(x) = e^{\int \cos x \, dx} = e^{\sin x}$

$e^{\sin x} y' + \cos x \, e^{\sin x} y = \cos x \, e^{\sin x}$

$\frac{d}{dx}\left(y e^{\sin x} \right) = \cos x \, e^{\sin x}$

$y e^{\sin x} = \int \cos x \, e^{\sin x} dx$

$= e^{\sin x} + C$

$y = 1 + C e^{-\sin x}$

(20 points) 2. The time rate of change of a rabbit population is inversely proportional to the square root of P. If at time $t = 0$ (in months) the population is 100 rabbits and is increasing at a rate of 15 rabbits a month, how many rabbits will there be after 1 year?

$$\frac{dP}{dt} = \frac{k}{\sqrt{P}} \qquad P(0) = 100$$

$$\frac{dP}{dt}(0) = 15 = \frac{k}{\sqrt{100}}$$

$$k = 150$$

$$\frac{dP}{dt} = \frac{150}{\sqrt{P}}$$

$$\int \sqrt{P}\, dP = \int 150\, dt$$

$$\frac{2}{3} P^{3/2} = 150t + C$$

$$\frac{2}{3}(100)^{3/2} = C$$

$$\frac{2}{3} P^{3/2} = 150t + \frac{2}{3}(100)^{3/2}$$

$$P^{3/2} = 225t + 1000$$

$$P(t) = (225t + 1000)^{2/3}$$

$$P(12) = (225 \cdot 12 + 1000)^{2/3}$$

$$= (2700 + 1000)^{2/3}$$

$$= (3700)^{2/3}$$

(15 points) 3. Solve the following linear system using matrices:

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & + & & + & x_4 & = & 0 \\ & & x_2 & + & 3x_3 & & & = & 0 \\ & & 3x_2 & + & x_3 & + & x_4 & = & 0 \end{array} \qquad (1)$$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 3 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{-3R_2+R_3} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & -8 & 1 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{8}R_3} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & -\frac{1}{8} \end{bmatrix}$$

$$\xrightarrow{-3R_3+R_2} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & +\frac{3}{8} \\ 0 & 0 & 1 & -\frac{1}{8} \end{bmatrix}$$

$$\xrightarrow{-2R_2+R_1} \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{8} \\ 0 & 0 & 1 & -\frac{1}{8} \end{bmatrix}$$

$\cancel{x_1 =}$ $x_4 = t$

$x_3 = \frac{1}{8}t$

$x_2 = -\frac{3}{8}t$

$x_1 = -\frac{1}{4}t$

Check: $-\frac{1}{4}t + 2(-\frac{3}{8}t) + t = 0$ ✓

$(-\frac{3}{8}t) + 3(\frac{1}{8}t) = 0$ ✓

$3(-\frac{3}{8}t) + \frac{1}{8}t + t = 0$ ✓

(15 points) 4. Compute the inverse to the 2×2 matrix $\begin{bmatrix} 4 & 7 \\ 2 & 5 \end{bmatrix}$

Two methods (both ok):

Formula: $\dfrac{1}{4\cdot 5 - 7\cdot 2}\begin{bmatrix} 5 & -7 \\ -2 & 4 \end{bmatrix}$

$= \dfrac{1}{6}\begin{bmatrix} 5 & -7 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 5/6 & -7/6 \\ -1/3 & 2/3 \end{bmatrix}$

Row Reduction:

$$\left[\begin{array}{cc|cc} 4 & 7 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1+R_2} \left[\begin{array}{cc|cc} 4 & 7 & 1 & 0 \\ 0 & \underbrace{5-\frac{7}{2}}_{3/2} & -\frac{1}{2} & 1 \end{array}\right]$$

$$\xrightarrow{\frac{2}{3}R_2} \left[\begin{array}{cc|cc} 4 & 7 & 1 & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{2}{3} \end{array}\right]$$

$$\xrightarrow{-7R_2+R_1} \left[\begin{array}{cc|cc} 4 & & \overbrace{1+\frac{7}{3}}^{10/3} & -14/3 \\ 0 & 1 & -\frac{1}{3} & 2/3 \end{array}\right]$$

$$\xrightarrow{\frac{1}{4}R_1} \left[\begin{array}{cc|cc} 1 & 0 & 5/6 & -7/6 \\ 0 & 1 & -\frac{1}{3} & 2/3 \end{array}\right]$$

$$\boxed{\begin{bmatrix} 5/6 & -7/6 \\ -1/3 & 2/3 \end{bmatrix}}$$

(15 points) 5. Let W be the set of vectors in $\mathbb{R}^4$ with $x_1 = 2x_2 = 3x_3$. Determine if W is a subspace of $\mathbb{R}^4$.

Closed under addition:

Let $\vec{u} = (u_1, u_2, u_3, u_4)$ be two vectors in W
+ $(v_1, v_2, v_3, v_4) = \vec{v}$

Then $(u_1+v_1, u_2+v_2, u_3+v_3, u_4+v_4) = \vec{u}+\vec{v}$
has first coordinate

$u_1 + v_1 = (2u_2) + (2v_2) = 2(u_2+v_2)$

since $\vec{u}$ is in W and $\vec{v}$ is in W.

also $= (3u_3) + (3v_3) = 3(u_3+v_3)$

Therefore $\vec{u}+\vec{v}$ is in W

Closed under Scalar multiplication:

Let $\vec{u} = (u_1, u_2, u_3, u_4)$ be in W

then, $c\vec{u} = (cu_1, c\cdot u_2, cu_3, cu_4)$
has first coordinate

$cu_1 = c(2u_2) = c(3u_3)$

since $\vec{u}$ is in W.

Therefore $cu_1 = 2(cu_2) = 3(cu_3)$

So $c\vec{u}$ is also in W.

(20 points) 6. Use determinants to determine if $\mathbf{v}_1 = (1,0,3,0)$, $\mathbf{v}_2 = (0,2,0,4)$ and $\mathbf{v}_3 = (1,2,0,4)$ are linearly independent.

$$[\vec{v}_1\ \vec{v}_2\ \vec{v}_3] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 0 & 0 \\ 0 & 4 & 4 \end{bmatrix}$$

has a 3X3 submatrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 0 & 0 \end{bmatrix}$ with determinant.

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 0 & 0 \end{vmatrix} = 3 \begin{vmatrix} 0 & 1 \\ 2 & 2 \end{vmatrix} = 3(-2 \cdot 1) = -6 \neq 0$$

Therefore $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are LI.