

Mathematics 1220      Exam III Review      Fall 2006

The material on this exam review reflects the concepts and techniques you should be familiar with for exam III. The review problems are categorized by topic, stated in the form you will see them on the test itself, and at roughly the same level as the test questions. Keep in mind that you will be asked to show all of your work for the exam, even though calculators *will* be allowed for this exam.

- Taylor and Maclaurin Series (§10.8)
  1. Write the Maclaurin series for  $f(x) = \sin x + \cos x$ . For what values of  $x$  does it represent  $f$ ?
  2. Find the first five terms of the Taylor series for  $e^x$  based at the point  $x = 2$ .
  3. Use the binomial series to write the Maclaurin series for  $f(x) = (1+x)^{1/2} + (1-x)^{1/2}$ .
  4. By writing  $\frac{1}{x} = \frac{1}{1-(1-x)}$  and using the known expansion of  $\frac{1}{1-x}$ , find the Taylor series for  $\frac{1}{x}$  in powers of  $x-1$ . Then, take the derivative of this series to find the Taylor expansion of  $\frac{1}{x^2}$ . For what values of  $x$  does it represent  $f$ ?
  5. Use the Taylor series for  $\frac{1}{x}$  based at 1 to find the Taylor series based at 1 for  $\ln x$ .
- Approximation by Taylor polynomials (§11.1)
  1. Find the Taylor polynomial of order 3 based at 2 for  $g(x) = x^3 - 2x^2 + 5x - 7$  and show that it is an exact representation of  $g(x)$ .
  2. Find the Taylor polynomial of order 4 based at 1 for  $f(x) = \frac{1}{x+1}$ , and use this polynomial to approximate  $f(1.2)$ . Then, obtain an expression for the error term  $R_4(x)$ , and find a reasonable upper bound for  $R_4(1.2)$ .
  3. Approximate  $\cos(0.3)$  with error less than 0.001.
  4. The Taylor series for  $\ln x$  in based at 1 is  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$ . Use the Taylor polynomial of order 5 based at 1 to approximate the integral  $\int_{0.8}^{1.2} \ln x \, dx$ .
- Numerical Integration (§11.2)
  1. Use the Trapezoid rule with  $n = 8$  to approximate  $\int_{0.8}^{1.2} \ln x \, dx$  and give a good upper bound for the error.
  2. Use the Parabolic rule/Simpson's rule with  $n = 8$  to approximate  $\int_{0.8}^{1.2} \ln x \, dx$  and give a good upper bound for the error.
  3. Determine an  $n$  so that the Trapezoid rule will approximate the integral  $\int_0^{0.5} e^{x^2} \, dx$  with an error  $E_n$  such that  $|E_n| \leq 10^{-3}$ . Then, approximate the integral with that  $n$  and the Trapezoid rule.
  4. Determine an  $n$  so that the Parabolic rule will approximate the integral  $\int_0^1 \frac{\sin x}{x} \, dx$  with an error  $E_n$  such that  $|E_n| \leq 10^{-5}$ . Then, approximate the integral with that  $n$  and the Parabolic rule.

- Numerical Equation Solving (§11.2-11.3)
  1. Use the Bisection method to approximate the positive root of  $f(x) = x - 2.5x(1 - x)$  on  $[\frac{1}{5}, 1]$  to two decimal places.
  2. Determine how many iterations of the Bisection method would be needed to approximate the root of a function  $f(x)$  on the interval  $[1, 2]$  with error less than  $10^{-10}$ .
  3. Use Newton's method to calculate  $\sqrt{2}$  to 5 decimal places.
  4. Use Newton's method to approximate any real root of the function  $f(x) = x^2 - 2x - \cos 3x$  to 3 decimal places.
  5. It is easy to see that the only root of the function  $f(x) = x^{1/3}$  is 0. However, Newton's method fails to converge to this root. Explain why.
  6. Use the Fixed Point algorithm approximate a fixed point of  $f(x) = 2.5x(1 - x)$  to 2 decimal places.
  7. Determine whether the fixed point algorithm can be applied to solve  $x = \tan x$  on the domain  $[4, 4.5]$  by checking all hypotheses of the fixed point theorem.
  8. Use the fixed point algorithm to approximate the fixed point of  $\tan^{-1} x + \pi$  to 3 decimal places.
- Approximating solutions to differential equations (§11.5)
  1. Use Euler's Method with  $h = 0.2$  to approximate the solution to the differential equation  $y' = xy$  with initial condition  $y(1) = 2$ , over the interval  $[1, 2]$ .
  2. Use Improved Euler's Method with  $h = 0.2$  to approximate the solution to the differential equation  $y' = xy$  with initial condition  $y(1) = 2$  over the interval  $[1, 2]$ .
  3. Apply Euler's Method with  $h = 1/N$ , where  $N$  is a positive integer, to the differential equation  $y' = y$ , with initial condition  $y(0) = 1$ . Derive the relationship  $y_n = y_0(1 + h)^n$ .
  4. Use Improved Euler's Method with  $h = 0.2$  to approximate the solution to the differential equation  $y' = 3y$  with initial condition  $y(0) = 2$  over the interval  $[0, 2]$ .

*Solutions will be posted within the next few days.*