Mathematics 1220 Exam I Review Fall 2006

The material on this exam review reflects the concepts and techniques you should be familiar with for Exam I, with the omission of material from sections 8.4 and 8.5. You should consider the homework for those sections ample review for the exam. The review problems are categorized by topic, stated in the form you will see them on the test itself, and at roughly the same level as the test questions. Keep in mind that you will be asked to show all your work for the exam.

- Transcendental Functions and their Properties (§7.1, 7.3-7.4, 7.7-7.8)

  1. Simplify the following expressions, as per the directions given:
     (a) Simplify using log and exp properties: $e^{x \ln x}$
     (b) Simplify using log and exp properties: $e^{\ln 3 + 2\ln x}$
     (c) Compute the limit $\lim_{x\to 0} (1+3x)^{1/x}$ (Hint: See Theorem A of §7.5)
     (d) Compute the limit $\lim_{n\to\infty} \left(1 - \frac{1}{2n}\right)^{6n}$
     (e) Rewrite $\sin(2\cos^{-1} x)$ as an expression in x which involves no trig functions.
     (f) Rewrite $\cos(\tan^{-1} x)$ as an expression in x which involves no trig functions.

  2. Compute the derivative $\frac{dy}{dx}$ for the following functions:
     (a) $y = \ln\sqrt{3x-2}$ (b) $y = 7^{(x+1)^2 - x}$ (c) $y = xe^{\cos(x^2)}$
     (d) $y = \sin^{-1}(\cos x)$ (e) $y = \sec x \tan^{-1}(e^x)$ (f) $y = \ln(\tan^2 x)$
     (g) $y = \sinh^{-1}\sqrt{x}$ (h) $y = e^{\tanh(e^x)}$ (i) $y = x^x$
     (j) Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = \frac{(x+3)^{1/3}(3x+2)^2}{\sqrt{x+1}}$

  3. Integrate the following functions (Hint: you shouldn't need to use more than a simple substitution and the antiderivative properties of transcendental functions)
     (a) $\int \frac{x^3+1}{2x^4+8x+1}\,dx$ (b) $\int \cot x\,dx$ (c) $\int \frac{e^{-1/x}}{x^2}\,dx$
     (d) $\int x 10^{x^2-1}\,dx$ (e) $\int \frac{1}{1+9x^2}\,dx$ (f) $\int \frac{x}{\sqrt{1-x^4}}\,dx$
     (g) $\int \text{sech}^2 x \tanh x\,dx$ (h) $\int \sinh x \sinh(\cosh x)\,dx$

- Inverse Functions and their Properties (§7.2)

  1. For the $f(x)$ given below, determine a domain where $f(x)$ is invertible and compute $f^{-1}(x)$ for $f(x)$ restricted to that domain.
     (a) $f(x) = x^2 + x - 6$ (b) $f(x) = \sqrt{\frac{1}{x-2}}$ (c) $f(x) = \frac{\cos x - 1}{\cos x + 1}$

  2. Use the Inverse Function Theorem to compute $(f^{-1})'(x)$ for the functions below. Note you will have to determine $f(x)$ in order to apply the theorem.
     (a) $f^{-1}(x) = \tan^{-1} x$ (b) $f^{-1}(x) = \ln x^3$ (c) $f^{-1}(x) = \text{csch}^{-1} x$

- Exponential Growth and Decay (§7.4)

  1. Suppose you are living in a dorm at college and you have observed that when one person on your floor gets sick, everybody on the floor gets sick. In fact, someone on your floor has strep throat right now, and you have a calculus test in a week! You are worried that you will feel so bad the day of the exam you won't be able to integrate anything.

     If you know it takes a million streptococcus bacteria to get you sick, you assume there's 10,000 bacteria in your system right now, and your biology major friend tells you that streptococcus populations have a doubling period of two days, will you be sick the day of the test (7 days from now)?

  2. Stewart wants to become a millionaire after 10 years by buying \$5,000 worth of a company's stock, which he will have to choose carefully to reach this goal. What must the sustained, annualized growth rate of the stock be in order to achieve his goal? Is Stewart being realistic?

  3. Newton's law of cooling states that the rate at which an object cools is proportional to the difference between the temperature $\theta(t)$ of the object and the constant ambient temperature T,

     $$\frac{d\theta}{dt} = -k(\theta - T),$$

     where $k > 0$ is a constant depending on the object. A corpse is discovered at 2 pm, and its temperature is found to be 85°F, with the ambient air temperature 68°F. Assuming $k = 0.5\ \text{hr}^{-1}$, find the time of death.

- First Order Linear Differential Equations (§7.6)

  1. Solve the following differential equations:

     (a) $\frac{dy}{dx} - \frac{y}{x} = xe^x$ (b) $\sin x \frac{dy}{dx} + 2y\cos x = \frac{1}{2}\sin 2x$ (c) $\frac{dy}{dx} + 2xy = x$

  2. A 100 gallon tank of brine intially contains 50 lbs of salt, completely dissolved. If 5 gallons of water per minute, containing 1 lbs of salt per gallon, is poured into the tank and well-stirred solution leaves the tank at the same rate, find a differential equation describing the rate of change of pounds of salt in the tank with respect to time (in minutes), then solve the differential equation. Use your solution to determine how long it will take for there to be 75 pounds of salt dissolved in the tank.

- Techniques of Integration (§8.1-8.5)

1. Integrate the following functions using the method of substitution:

   (a) $\displaystyle\int \frac{\sin(2t)}{1-\sin^2(2t)}\,dt$ (b) $\displaystyle\int \frac{\sec^2(\ln x)}{2x}\,dx$ (c) $\displaystyle\int \frac{xe^{\sqrt{2x^2+3}}}{\sqrt{2x^2+3}}\,dx$

2. Integrate the following functions using appropriate trigonometric techniques:

   (a) $\displaystyle\int \sin^{3/5}(2x+1)\cos^3(2x+1)\,dx$ (b) $\displaystyle\int \cos^4 x\,dx$

   (c) $\displaystyle\int_{-\pi}^{\pi} \cos mx \cos nx\,dx$, where m and n are integers and $m \neq n$.

3. Integrate the following functions using rationalizing substitutions:

   (a) $\displaystyle\int x^2(x+1)^{3/2}\,dx$ (b) $\displaystyle\int \frac{2x-1}{x^2-6x+18}\,dx$ (c) $\displaystyle\int \sqrt{5-4x-x^2}\,dx$

Solutions will be posted within the next few days.