

Mathematics 1220 Final Exam Review Fall 2006

The material on this exam review reflects the major concepts and techniques of the course.

Be aware that material from sections of the text not covered explicitly on this review may be used implicitly in individual problems. For example, I may not ask you directly to use L'Hopital's rule to compute a limit, but this technique may be necessary to compute an improper integral.

The review problems are categorized by chapter, stated in the form you will see them on the test itself, and at roughly the same level as the test questions. Keep in mind that you will be asked to show all your work for the exam.

- Chapter 7: Transcendental Functions and First Order Differential Equations.

1. Compute the derivative $\frac{dy}{dx}$ for the following functions:

(a) $y = 7^{(x+1)^2-x}$ (b) $y = \sin^{-1}(\cos x)$ (c) $y = e^{\tanh(e^x)}$ (d) $y = x^x$

2. Integrate the following functions (Hint: you shouldn't need to use more than a simple substitution and the antiderivative properties of transcendental functions)

(a) $\int \cot x \, dx$ (b) $\int x10^{x^2-1} \, dx$ (c) $\int \frac{1}{1+9x^2} \, dx$ (d) $\int \operatorname{sech}^2 x \tanh x \, dx$

3. Suppose you are living in a dorm at college and you have observed that when one person on your floor gets sick, everybody on the floor gets sick. In fact, someone on your floor has strep throat right now, and you have a calculus final in a week! You are worried that you will feel so bad the day of the exam you won't be able to integrate anything.

If you know it takes a million streptococcus bacteria to get you sick, you assume there's 10,000 bacteria in your system right now, and your biology major friend tells you that streptococcus populations have a doubling period of two days, how many days will it be until you get sick? You may leave your answer in terms of natural log.

4. Solve the following differential equations:

(a) $\frac{dy}{dx} - \frac{y}{x} = xe^x$ (b) $\frac{dy}{dx} + 2xy = x$

- Chapter 8: Techniques of Integration

Integrate the following functions using any appropriate technique.

1. $\int \sin^{3/5}(2x+1) \cos^3(2x+1) \, dx$ 2. $\int \cos^4 x \, dx$ 3. $\int \frac{2x-1}{x^2-6x+18} \, dx$

4. $\int \sqrt{5-4x-x^2} \, dx$ 5. $\int \ln(x^2+4) \, dx$ 6. $\int x \cot^2 x \, dx$

7. $\int \frac{x+9}{x^3+9x} \, dx$ 8. $\int \frac{x^4}{1-x^2} \, dx$ 9. $\int e^{x/3} \sin 3x \, dx$

- Chapter 9: Improper Integrals

Determine whether the following improper integrals converge or diverge. Be sure to justify your answer completely, but note that you do not necessarily need to (or are able to) compute the integrals in order to prove convergence.

$$1. \int_1^{\infty} \frac{\ln x}{\sqrt{x^3 + 2x + 1}} dx \quad 2. \int_0^{\infty} e^{-x} \cos x dx \quad 3. \int_{\pi/3}^{\pi/2} \frac{\tan x}{(\ln \cos x)^2} dx$$

- Chapter 10: Power Series Representations of Functions

1. Determine whether the following infinite series converge or diverge. Be sure to justify your answers completely, but note you do not need to compute the sum. If the series is not positive, you must state whether the convergence is absolute or conditional.

$$(a) \sum_{n=0}^{\infty} e^{-2n} \quad (b) \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}} \right) \quad (c) \sum_{k=1}^{\infty} \frac{1}{16,000k}$$

$$(d) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\pi}} \quad (e) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{2/3}} \quad (f) \sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$$

2. Find the convergence set for the following power series. If applicable, do not forget to check the endpoints.

$$(a) \sum_{n=0}^{\infty} \frac{n!(x+1)^n}{3^n} \quad (b) \sum_{n=0}^{\infty} \frac{x^n}{n^3 + 1} \quad (c) \sum_{n=0}^{\infty} \frac{(-2)^{n+1} x^n}{(2n)!}$$

3. Use power series operations and known power series to find power series representations for the following functions. Then, determine the interval of convergence for the series.

$$(a) f(x) = \frac{1}{(1+x)^2} \quad (b) f(x) = \int_0^x \frac{\tan^{-1} t}{t} dt \quad (c) f(x) = xe^{-x}$$

4. By writing $\frac{1}{x} = \frac{1}{1-(1-x)}$ and using the known expansion of $\frac{1}{1-x}$, find the Taylor series for $\frac{1}{x}$ in powers of $x-1$. Then, take the derivative of this series to find the Taylor expansion of $\frac{1}{x^2}$. For what values of x does it represent f ?
5. Use the Taylor series for $\frac{1}{x}$ based at 1 to find the Taylor series based at 1 for $\ln x$.

- Chapter 11: Error in Numerical Techniques

1. Approximate $\cos(0.3)$ with error less than 0.001.
2. Determine an n so that the Trapezoid rule will approximate the integral $\int_0^{0.5} e^{x^2} dx$ with an error E_n such that $|E_n| \leq 10^{-3}$. Then, approximate the integral with that n and the Trapezoid rule.
3. Determine an n so that the Parabolic rule will approximate the integral $\int_0^1 \frac{\sin x}{x} dx$ with an error E_n such that $|E_n| \leq 10^{-5}$. Then, approximate the integral with that n and the Parabolic rule.
4. Determine how many iterations of the Bisection method would be needed to approximate the root of a function $f(x)$ on the interval $[1, 2]$ with error less than 10^{-10} .

5. Use Newton's method to approximate any real root of the function $f(x) = x^2 - 2x - \cos 3x$ to 3 decimal places.
 6. Use the fixed point algorithm to approximate the fixed point of $\tan^{-1} x + \pi$ to 3 decimal places.
- Chapter 12: Conic Sections and Polar Coordinates
 1. Name the conic that has each equation. Determine the foci and directrices in each case.
 - (a) $y^2 = 4x$
 - (b) $\frac{x^2}{9} + \frac{y^2}{4} = 1$
 - (c) $10x^2 - 25y^2 = 100$
 2. Write the following conic section equations in standard form. State the angle of rotation of the axes, the center, and the conic section this equation represents.
 - (a) $-\frac{1}{2}x^2 + 7xy - \frac{1}{2}y^2 - 6\sqrt{2}x - 6\sqrt{2}y = 0$
 - (b) $x^2 + xy + y^2 = 6$
 - (c) $4x^2 + 2\sqrt{3}xy + 2y^2 + 10\sqrt{3}x + 10y = 5$
 - Chapter 18: Second Order Linear Differential Equations
 1. Find the general solution to the following second order homogeneous linear differential equations.
 - (a) $y'' - y = 0$
 - (b) $y'' - 2y' + y = 0$
 - (c) $y'' + 2y' + 2y = 0$
 2. Find a particular solution to the following nonhomogeneous equations.
 - (a) $y'' + 6y' + 25y = 17 \cos x$
 - (b) $y'' - 4y = e^{2x}$
 - (c) $y'' + 2y' + 2y = x^2 + 1$

The solutions should be posted in a couple of days