1. Evaluate

(a) $\int (5x^3(x^4-1)^{-2/3})\,dx = \frac{5}{4}\int (x^4-1)^{-2/3} \cdot 4x^3 dx$

$u = x^4 - 1$

$du = 4x^3$

$= \frac{5}{4}\int u^{-2/3} du = \frac{5}{4} \cdot 3u^{1/3} + C$

$= \boxed{\frac{15}{4} u^{1/3} + C}$

(b) $\int \left(3\sqrt[3]{t} - \frac{4}{t^3} + 5t^2 - \cos t + 2\right) dt = \int (3t^{1/3} - 4t^{-3} + 5t^2 - \cos t + 2)\,dt$

$= 3 \cdot \frac{3}{4} t^{4/3} + 4 \cdot \frac{1}{2} t^{-2} + \frac{5}{3} t^3 - \sin t + 2t + C$

$= \boxed{\frac{9}{4} t^{4/3} + 2t^{-2} + \frac{5}{3} t^3 - \sin t + 2t + C}$

2. Solve the following differential equation.

$\frac{dy}{dx} = \frac{(x^2 - \sqrt{x})}{2y^3}$ such that $y = -1$ when $x = 1$

$2y^3 dy = (x^2 - x^{1/2})dx \Rightarrow 2\int y^3 dy = \int (x^2 - x^{1/2})dx$

$\Rightarrow \frac{1}{2} y^4 = \frac{1}{3}x^3 - \frac{2}{3}x^{3/2} + C_1 \Rightarrow y^4 = \frac{2}{3}x^3 - \frac{4}{3}x^{3/2} + C_2$

We know $(-1)^4 = \frac{2}{3} - \frac{4}{3} + C_2 \Rightarrow C_2 = 1 + \frac{2}{3} = \frac{5}{3}$,

so $y^4 = \frac{2}{3}x^3 - \frac{4}{3}x^{3/2} + \frac{5}{3}$

$y = \left(\frac{2}{3}x^3 - \frac{4}{3}x^{3/2} + \frac{5}{3}\right)^{1/4}$

3. For $f(x)=\dfrac{x^2-2x-2}{x-3}$

Find the asymptotes, if they exist.

vertical: $x=3$

oblique: $\lim\limits_{x\to\pm\infty} \dfrac{x^2-2x-2}{x-3}\cdot\dfrac{\frac{1}{x}}{\frac{1}{x}}$

$=\lim\limits_{x\to\pm\infty} \dfrac{x-2-\frac{2}{x}}{1-\frac{3}{x}} = \lim\limits_{x\to\pm\infty} x-2$

Vertical asymptote(s): $x=3$

Oblique/Horizontal asymptote(s): $y=x-2$

4. For the function $f(x) = \dfrac{4x-1}{x-4}$ on the closed interval [-1, 3], decide whether or not the Mean Value Theorem for Derivatives applies. If it does, find all possible values of c. If not, then state the reason.

MVT applies: True or False

$$\frac{f(3)-f(-1)}{3-(-1)} = \frac{\frac{12-1}{3-4} - \frac{-4-1}{-5}}{4} = \frac{-11+1}{4} = -3$$

$$f'(x) = \frac{4(x-4)-(4x-1)}{(x-4)^2} = \frac{4x-16-4x+1}{(x-4)^2} = \frac{-15}{(x-4)^2}$$

Solve $\dfrac{-15}{(x-4)^2} = -3 \Rightarrow (x-4)^2 = 5$

$\Rightarrow x - 4 = \pm\sqrt{5}$

$\Rightarrow \boxed{x = -4+\sqrt{5}}$ or $x = -4-\sqrt{5} \notin [-1,3]$

If true, then c = $-4+\sqrt{5}$

If false, then why? ______________________

5. Evaluate this integral. $\displaystyle\int \frac{(2x+3)^2}{\sqrt{x}}\,dx = \int \frac{4x^2+12x+9}{\sqrt{x}}\,dx$

$$= \int \left(4x^{3/2} + 12x^{1/2} + 9x^{-1/2}\right) dx$$

$$= 4 \cdot \frac{2}{5} x^{5/2} + 12 \cdot \frac{2}{3} x^{3/2} + 9 \cdot 2x^{1/2} + C$$

$$= \boxed{\frac{8}{5} x^{5/2} + 8x^{3/2} + 18x^{1/2} + C}$$

6. Evaluate the definite integral **using the definition** (the tedious way).

$$\int_0^2 3x^2\,dx \quad .$$

$$\Delta x = \frac{2-0}{n} = \frac{2}{n}$$

$$x_i = 0 + i\Delta x = \frac{2i}{n}$$

$$\int_0^2 3x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} 3x_i^2\,\Delta x = \lim_{n\to\infty} 3 \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} 3 \sum \frac{4i^2}{n^2} \cdot \frac{2}{n} = \lim_{n\to\infty} \frac{24}{n^3} \sum i^2$$

$$= \lim_{n\to\infty} \frac{24}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n\to\infty} 4 \cdot \frac{(n+1)(2n+1)}{n^2}$$

$$= \lim_{n\to\infty} 4 \cdot \frac{2n^2+3n+1}{n^2}$$

$$= \lim_{n\to\infty} 4 \cdot \left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$$

$$= \boxed{8}$$

7. (a) Find $\frac{dG}{dx}$, when $G(x)=\int_1^{x^2+x}\sqrt{2z+\sin z}\,dz$.

$$u = x^2+x \Rightarrow G(x) = \int_1^u \sqrt{2z+\sin z}\,dz$$

$$\frac{dG}{dx} = \frac{dG}{du}\cdot\frac{du}{dx} = \left(\sqrt{2u+\sin u}\right)\cdot(2x+1)$$

$$= (2x+1)\sqrt{2(x^2+x)+\sin(x^2+x)}$$

(b) Find $\frac{dG}{dx}$, when $G(x)=\int_{\cos(x)}^{\sin(x)} t^5\,dt = \int_0^{\sin x} t^5\,dt + \int_{\cos x}^0 t^5\,dt$

$$= \underbrace{\int_0^{\sin x} t^5\,dt}_{A(x)} - \underbrace{\int_0^{\cos x} t^5\,dt}_{B(x)}$$

$$u = \sin(x) \Rightarrow A(x) = \int_0^u t^5\,dt$$

$$\Rightarrow \frac{dA}{dx} = \frac{dA}{du}\cdot\frac{du}{dx} = u^5\cos(x) = \sin^5(x)\cos(x)$$

$$v = \cos(x) \Rightarrow B(x) = \int_0^v t^5\,dt$$

$$\Rightarrow \frac{dB}{dx} = \frac{dB}{dv}\cdot\frac{dv}{dx} = -v^5\sin(x) = -\sin(x)\cos^5(x)$$

$$\frac{dG}{dx} = \frac{dA}{dx} - \frac{dB}{dx} = \sin^5(x)\cos(x) + \sin(x)\cos^5(x)$$

8. Find the average value of f(x) on the interval $[0, 2\pi]$, where $f(x) = \dfrac{\sin x \cos x}{\sqrt{1 + \cos^2 x}}$.

$$\text{average} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\sin x \cos x}{\sqrt{1+\cos^2 x}}\, dx$$

$u = 1 + \cos^2 x$

$du = 2 \sin x \cos x \, dx$

$$\text{average} = \frac{1}{2\pi} \cdot \frac{1}{2} \int_0^{2\pi} \frac{2 \sin x \cos x \, dx}{\sqrt{1 + \cos^2 x}}$$

$$= \frac{1}{4\pi} \int_{x=0}^{2\pi} u^{-1/2}\, du = \frac{1}{4\pi} \left[2u^{1/2} \right]_{x=0}^{2\pi}$$

$$= \frac{1}{2\pi} \left[(1 + \cos^2 x)^{1/2} \right]_{x=0}^{2\pi} = \frac{1}{2\pi} \left(2^{1/2} - 2^{1/2} \right)$$

$$= \boxed{0}$$

9. Solve $x^4 - 53 = 0$ using Newton's Method, accurate to five decimal places.

$$f(x) = x^4 - 53$$

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)} = X_n - \frac{X_n^4 - 53}{4X_n^3}$$

Lets use $X_1 = 1$

n	X_n
1	1
2	14
3	10.50482872
4	7.89005161
5	5.94451463
6	4.52146234
7	3.53444063
8	2.95092205
9	2.72882687
10	2.69868071
11	2.69816802
12	2.$\underline{69816}$788

$$\boxed{\approx 2.69817}$$