1) a)

$$\lim_{x \to 0} \frac{\sin 5x - \sin 3x}{\tan 3x} = \lim_{x \to 0} \frac{\sin 5x - \sin 3x}{\sin 3x} \cdot \cos 3x$$

$$= \lim_{x \to 0} \left(\frac{\sin 5x}{\sin 3x} - \frac{\sin 3x}{\sin 3x} \right) \cdot \underbrace{\cos 3x}_{1}$$

$$= \lim_{x \to 0} \left(\frac{\sin 5x}{5x} \cdot \frac{3x}{\sin 3x} \cdot \frac{5}{3} - 1 \right) \cdot 1$$

$$= \left(\frac{5}{3} - 1 \right) \cdot 1$$

$$= \boxed{\frac{2}{3}}$$

b)

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - x - 2} = \lim_{x \to 2} \frac{(x-2)(x+3)}{(x-2)(x+1)}$$

$$= \lim_{x \to 2} \frac{x+3}{x+1} = \boxed{\frac{5}{3}}$$

1) c) $\lim_{x \to \infty} \dfrac{3 - 2x + x^2 - 6x^3}{2x^3 + 5x - x^2 + \sqrt{3x^3}} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$= \lim_{x \to \infty} \dfrac{\frac{3}{x^3} - \frac{2}{x^2} + \frac{1}{x} - 6}{2 + \frac{5}{x^2} - \frac{1}{x} + \sqrt{3}\,\underbrace{x^{3/2} \cdot \frac{1}{x^3}}_{\frac{1}{x^{3/2}}}}$

$= \dfrac{-6}{2} = \boxed{-3}$

d) $\lim_{x \to -2^-} \dfrac{1 - 4x^2}{x^2 - 4} = \lim_{x \to -2^-} \dfrac{1 - 4x^2}{(x-2)(x+2)}$

$= \dfrac{(-)}{(-)\,(-)} \cdot \infty = \boxed{-\infty}$

2)

$$f'(x) = 17\left(6x^5 - x^4 + 3\sqrt{2x} - \frac{3}{4x} + 101\right)^{16} \cdot \left(30x^4 - 4x^3 + 3\sqrt{2}\cdot\frac{1}{2}x^{-1/2} + \frac{3}{4}x^{-2}\right)$$

$$g'(x) = \frac{1}{2}\left(\sin^3 x + \cos(4x+1)\right)^{-1/2} \cdot \cancel{\left(3\sin^2 x \cos x\right.} \cdot \left(3\sin^2 x \cos x - \sin(4x+1)\cdot 4\right)$$

$$h'(x) = \cos(\cos(\sin^2(3x-5)))\cdot\left(-\sin(\sin^2(3x-5))\right) \cdot (2\sin(3x-5))\cdot(\cos(3x-5))\cdot 3$$

$$s'(x) = \frac{(\sin 2x \ \sin 3x)'(x+7) - (\sin 2x \sin 3x)(x+7)'}{(x+7)^2}$$

$$= \frac{\left((\sin 2x)'(\sin 3x) + (\sin 2x)(\sin 3x)'\right)(x+7) - \sin 2x \sin 3x}{(x+7)^2}$$

$$= \frac{(2\cos 2x \sin 3x + 3\sin 2x \cos 3x)(x+7) - \sin 2x \sin 3x}{(x+7)^2}$$

3)

$$\int \sqrt{ax+b}\,dx = \frac{1}{a}\int u^{1/2}\,du = \boxed{\frac{1}{a}\cdot\frac{2}{3}u^{3/2} + (\text{constant})}$$

$$\boxed{\begin{array}{l} \text{Let } u = ax+b \\ \quad du = a\,dx \end{array}}$$

$$\int \sin^2(x^3-2)\,x^2\cos(x^3-2)\,dx = \frac{1}{3}\int u^2\,du$$

$$\boxed{\begin{array}{l} \text{Let } u = \sin(x^3-2) \\ \quad du = 3x^2\cos(x^3-2)\,dx \end{array}}$$

$$= \boxed{\frac{1}{3}\cdot\frac{1}{3}u^3 + C}$$

constant

$$= \boxed{\frac{1}{9}\sin^3(x^3-2) + C}$$

$$\int_1^2 \frac{2x-1}{(5x^2-5x)^3}\,dx = \int_1^2 \frac{1}{5^3}\cdot\frac{2x-1}{(5x^2-5x)^3}\,dx$$

$$\boxed{\begin{array}{l} \text{Let } u = x^2-x \\ \quad du = (2x-1)\,dx \end{array}}$$

$$= \frac{1}{5^3}\int_1^2 u^{-3}\,dx = \frac{1}{5^3}\left[-\frac{1}{2}u^{-2}\right]_{x=1}^{2}$$

$$= -\frac{1}{2\cdot 5^3}\left[(x^2-x)^{-2}\right]_1^2$$

$$= -\frac{1}{2\cdot 5^3}\left(2^{-2}-0\right)$$

$$= -\frac{1}{2\cdot 5^3}\cdot\frac{1}{2^2} = -\frac{1}{2^3\cdot 5^3} = \boxed{-\frac{1}{1000}}$$

Revised problem

Solve

$$\int_2^3 \frac{2x-1}{(5x^2-5x)^3}\,dx$$

Answer:

$$\int_2^3 \frac{2x-1}{(5x^2-5x)^3}\,dx = \int_2^3 \frac{1}{5^3}\cdot\frac{2x-1}{(x^2-x)^3}\,dx$$

$$\boxed{\begin{array}{l}\text{Let } u = x^2 - x \\ du = (2x-1)\,dx\end{array}}$$

$$= \frac{1}{5^3}\int_2^3 u^{-3}\,du$$

$$= \frac{1}{5^3}\left[-\frac{1}{2}u^{-2}\right]_{x=2}^{3}$$

$$= -\frac{1}{2\cdot 5^3}\left[(x^2-x)^{-2}\right]_2^3$$

$$= -\frac{1}{2\cdot 5^3}\left(6^{-2} - 2^{-2}\right)$$

$$= \boxed{-\frac{1}{2\cdot 5^3}\left(\frac{1}{36} - \frac{1}{4}\right)}$$

$$\int_0^1 \frac{2x^3 - 4x + 5\sqrt{x}}{\sqrt{x}}\,dx = \int_0^1 (2x^{5/2} - 4x^{1/2} + 5)\,dx$$

$$= \left[2 \cdot \frac{2}{7} x^{7/2} - 4 \cdot \frac{2}{3} x^{3/2} + 5x\right]_0^1$$

$$= \boxed{\frac{4}{7} - \frac{8}{3} + 5}$$

4)

$$\frac{d}{dx}\left(\sin(xy) - y^2\right) = \frac{d}{dx}\left(2x^3+1\right)$$

$$\cos(xy)\cdot(xy)' - 2yy' = 6x^2$$

$$\cos(xy)(xy'+y) - 2yy' = 6x^2$$

$$y'x\cos(xy) - y'\cdot 2y + y\cos(xy) = 6x^2$$

$$y'(x\cos(xy) - 2y) = 6x^2 - y\cos(xy)$$

$$\boxed{y' = \frac{dy}{dx} = \frac{6x^2 - y\cos(xy)}{x\cos(xy) - 2y}}$$

5) a)

$$\text{Average} = \frac{1}{3}\int_0^3 x^2\,dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_0^3 = \frac{3^3}{3^2} = 3$$

Solve $f(c) = 3 \Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$

Only $\boxed{c = \sqrt{3}}$ is in $[0,3]$

b)

$$\text{Average slope} = \frac{f(3) - f(0)}{3 - 0} = \frac{14 - 5}{3} = \frac{9}{3} = 3$$

$f'(x) = x^2$

Solve $f'(c) = 3 \Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$

Only $\boxed{c = \sqrt{3}}$ is in $[0,3]$

6)

$$\frac{dx}{dt} = 2t^5$$

$$\frac{dy}{dt} = t^8$$

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 \sqrt{4t^{10} + t^{16}}\, dt$$

$$= \int_0^1 \sqrt{t^{10}(4+t^6)}\, dt = \int_0^1 t^5\sqrt{(4+t^6)}\, dt$$

$$\boxed{\begin{array}{l} \text{Let } u = 4 + t^6 \\ du = 6t^5 dt \end{array}}$$

$$= \frac{1}{6}\int_{t=0}^{1} u^{1/2}\, du$$

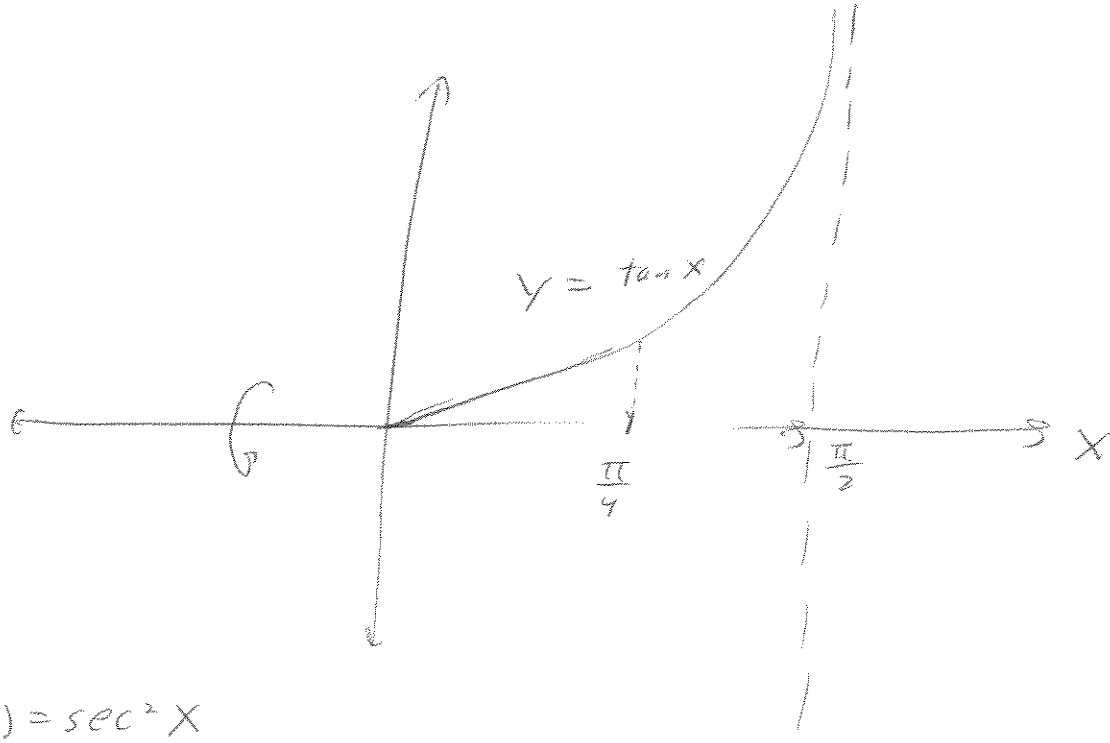
$$= \frac{1}{6}\cdot\frac{2}{3}\cdot\left[u^{3/2}\right]_{t=0}^{1}$$

$$\cancel{= \frac{1}{6}\cdot\frac{2}{3} = \boxed{\frac{1}{9}}}$$

$$= \frac{1}{6}\cdot\frac{2}{3}\cdot\left[(4+t^6)^{3/2}\right]_0^1$$

$$= \frac{1}{9}\cdot\left(5^{3/2} - 4^{3/2}\right) = \boxed{\frac{1}{9}\left(5\sqrt{5} - 8\right)}$$

7)

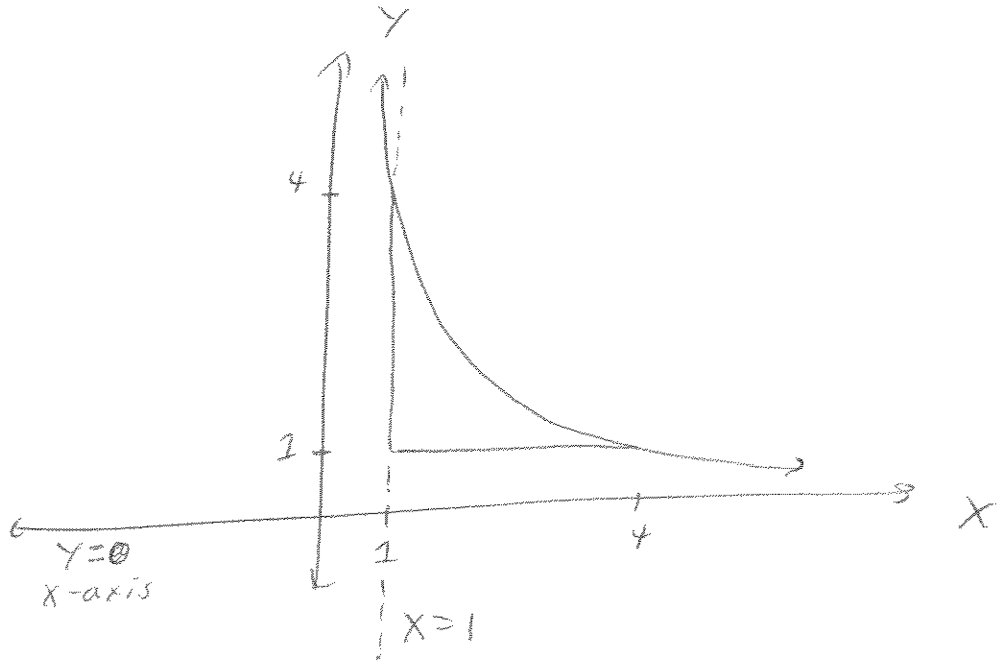


$f'(x) = \sec^2 x$

$$A = \int_0^{\pi/4} 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

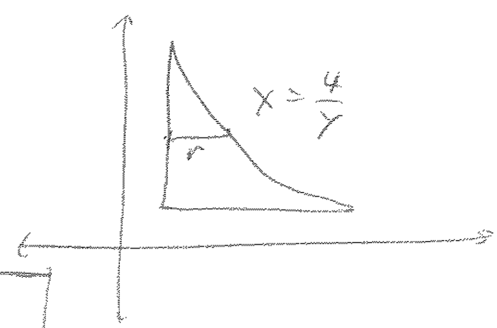
$$\boxed{= \int_0^{\pi/4} 2\pi \tan(x)\sqrt{1+\sec^4(x)}\,dx}$$

8)



a) <u>Disk Method</u>

$r = \frac{4}{y} - 1$

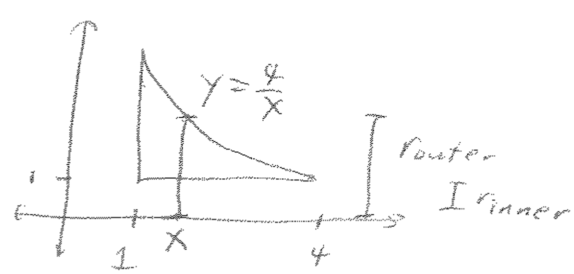


$$\boxed{V = \int_1^4 \cancel{2}\pi\left(\frac{4}{y} - 1\right)^2 dy}$$

b) <u>Washer Method</u>

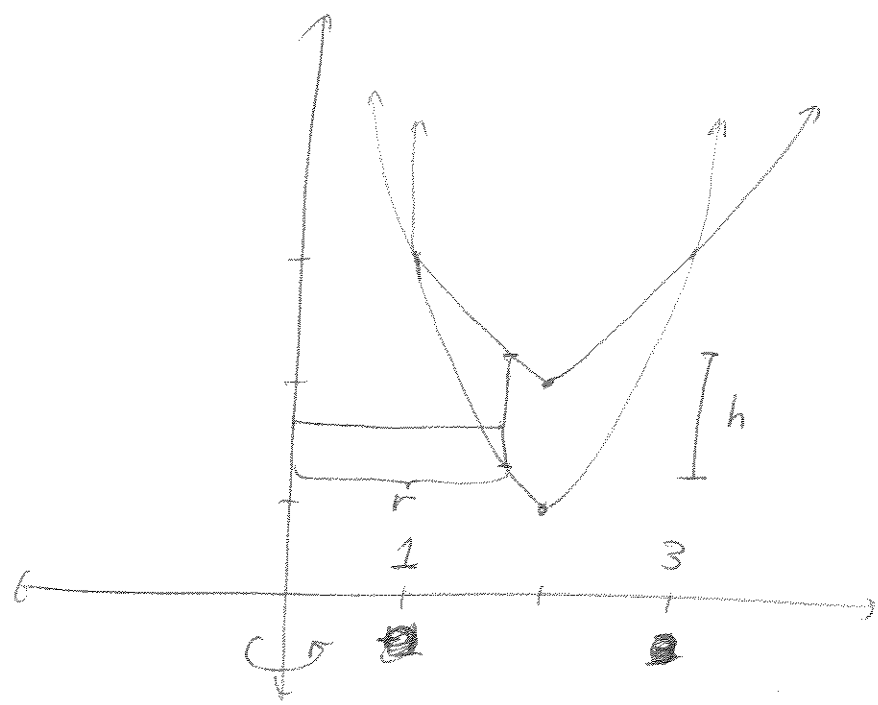
$r_{outer} = \frac{4}{x}$

$r_{inner} = 1$



$$\boxed{V = \cancel{2}\pi \int_1^4 \left(\left(\frac{4}{x}\right)^2 - 1^2\right) dx}$$

9)



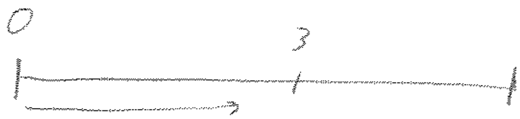
Shell Method

$$V = 2\pi \int_a^b r h \, dx$$

$$\boxed{V = 2\pi \int_1^3 x \left(\left[(x-2)^2 + 2 \right] - \left[2(x-2)^2 + 1 \right] \right) dx}$$

10)

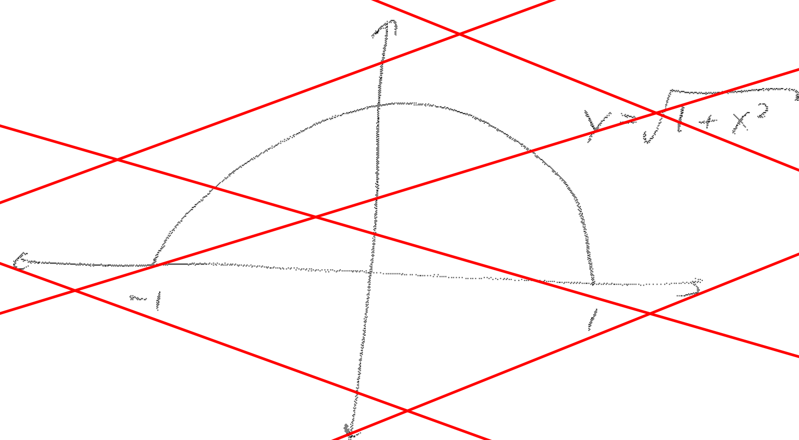
$$F = kx \Rightarrow 6 = k \cdot 1 \Rightarrow k = 6$$



$$W = \int_0^3 6x\,dx = \left[6 \cdot \frac{x^2}{2}\right]_0^3 = 3\left[x^2\right]_0^3 = \boxed{27}$$

ft-pounds

11)

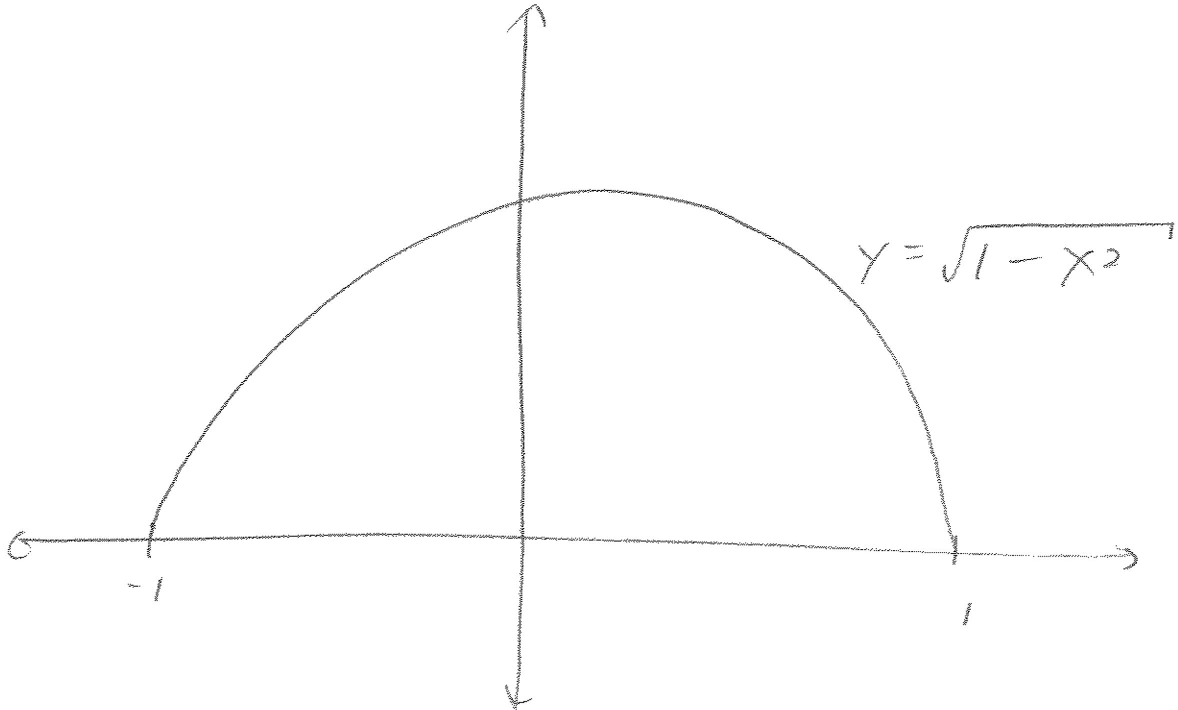


By symmetry, $\bar{x} = 0$.

$$\bar{y} = \frac{M_x}{m} = \frac{\frac{1}{2}\int_{-1}^{1}\left[\left(\sqrt{1+x^2}\right)^2 - 0^2\right]dx}{\int_{-1}^{1}\sqrt{1+x^2}\,dx}$$

You can't do this integral with what you know, but notice that Area of semicircle $= \frac{1}{2}\pi r^2$ and $r = 1$.

11)



By symmetry, $\bar{x} = 0$.

$$\bar{y} = \frac{M_x}{m} = \frac{\frac{1}{2}\int_{-1}^{1}\left[(\sqrt{1-x^2})^2 - 0^2\right]dx}{\underbrace{\int_{-1}^{1}\sqrt{1-x^2}\,dx}}$$

You can't do this integral with what you know, but notice that the area of a semicircle is $\frac{1}{2}\pi r^2$, and in this case $r=1$.

So,

$$\bar{y} = \frac{\frac{1}{2}\int_{-1}^{1} (1-x^2)\,dx}{\frac{1}{2}\pi} = \frac{1}{\pi}\left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

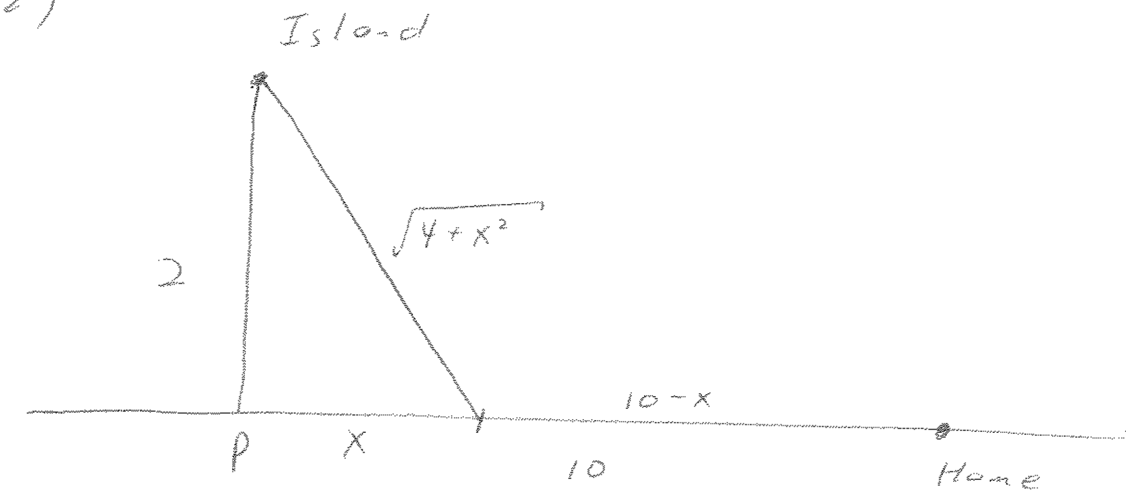
$$= \frac{1}{\pi}\left[1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{2}{3} - \left(-\frac{2}{3}\right)\right]$$

$$= \frac{4}{3\pi}$$

Centroid $(\bar{x}, \bar{y}) = \left(0, \frac{4}{3\pi}\right)$

12)



$$T = \frac{\sqrt{4+x^2}}{3} + \frac{10-x}{7}$$

$$\frac{dT}{dx} = \frac{1}{3}\cdot\frac{1}{2}(4+x^2)^{-1/2}\cdot 2x - \frac{1}{7}$$

$$0 = \frac{1}{3}\frac{x}{\sqrt{4+x^2}} - \frac{1}{7} \Rightarrow \frac{x}{\sqrt{4+x^2}} = \frac{3}{7}$$

$$\Rightarrow \frac{x^2}{4+x^2} = \frac{9}{49} \Rightarrow \frac{49}{9}x^2 = 4+x^2 \Rightarrow \frac{40}{9}x^2 = 4$$

$$\Rightarrow x^2 = \frac{9}{10} \Rightarrow \boxed{x = \sqrt{\frac{9}{10}} \text{ miles downstream from } P}$$

(I won't force you to verify that $\frac{d^2T}{dx^2} > 0$. You can apply common sense.)

13) $f(x) = \dfrac{(x+2)(2x+1)}{x^2} = \dfrac{2x^2+5x+2}{x^2}$

$= 2 + 5x^{-1} + 2x^{-2}$

a) Vertical: $\boxed{x = 0}$

Horizontal: $\lim\limits_{x\to\infty} 2 + 5x^{-1} + 2x^{-2} = 2$

$\boxed{y = 2}$

b) $f'(x) = -5x^{-2} - 4x^{-3} = -x^{-3}(5x+4)$

$f'(x) = -(-)(-)$ < 0	$f'(x) = -(-)(+)$ > 0	$f'(x) = -(+)(+)$ < 0
decr ↓	$-\frac{4}{5}$ incr ↑	0 decr ↓

Increasing: $(-\frac{4}{5}, 0)$

Decreasing: $(-\infty, -\frac{4}{5}) \cup (0, \infty)$

c) Local min at $x = -\frac{4}{5}$

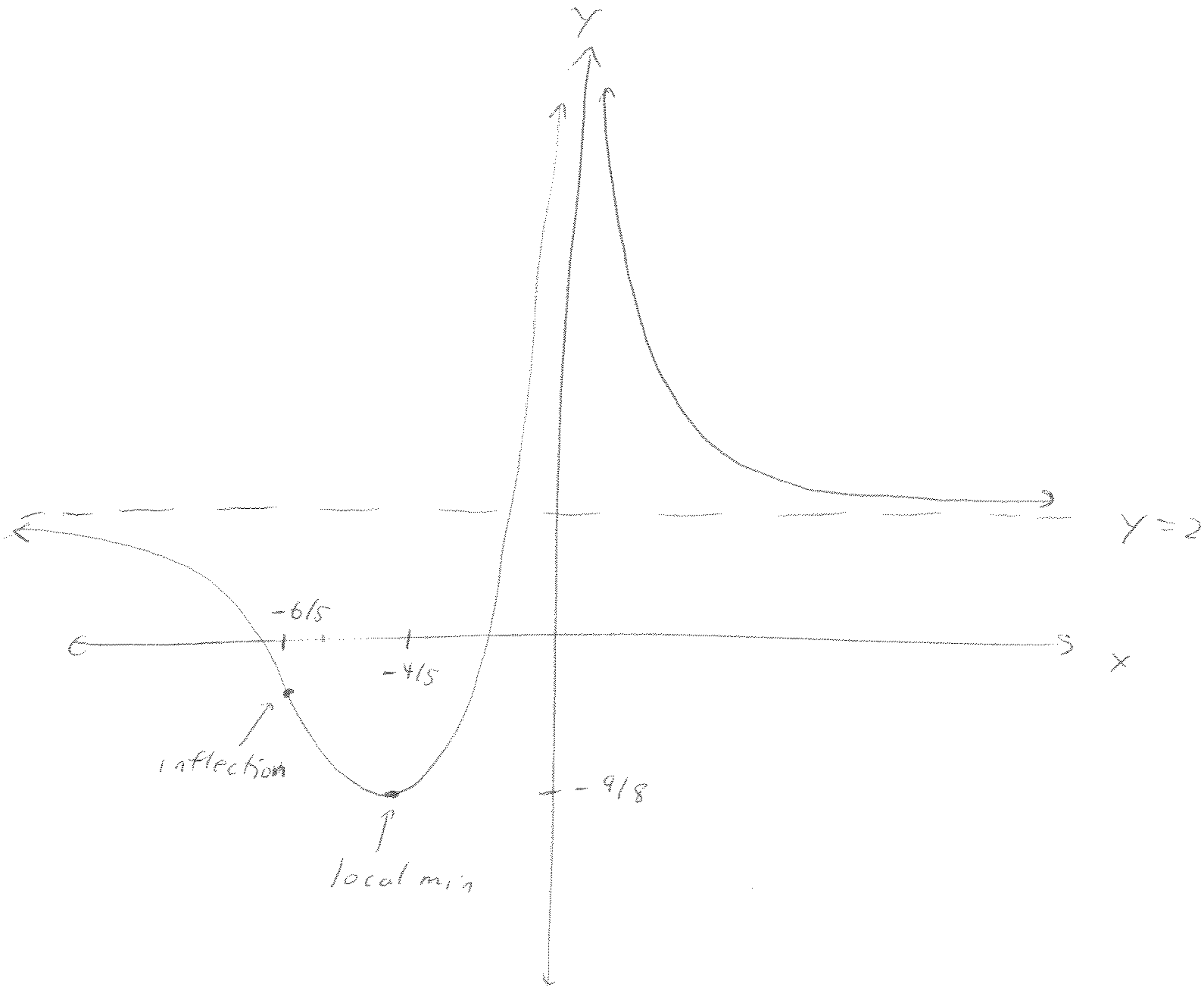
$$f\left(-\frac{4}{5}\right) = \frac{\left(-\frac{4}{5}+2\right)\left(-2\cdot\frac{4}{5}+1\right)}{\left(\frac{4}{5}\right)^2} = \frac{\frac{6}{5}\cdot\frac{-3}{5}}{\left(\frac{4}{5}\right)^2} = \frac{-6\cdot 3}{4\cdot 4} = -\frac{9}{8}$$

Local min: $\boxed{\left(-\frac{4}{5}, -\frac{9}{8}\right)}$

d) $f''(x) = 10x^{-3} + 12x^{-4} = 2x^{-4}(5x+6)$

$f''(x) = (+)(-) < 0$	$f''(x) = (+)(+) > 0$	$f''(x) = (+)(+) > 0$
down ↓	$-\frac{6}{5}$ ↑ up	0 up ↑

e) Inflection point at $\boxed{x = -\frac{6}{5}}$



Y
Y = 2
-6/5
-4/5
X
inflection
local min
-9/8