

Math 1210, Calculus I

SAMPLE FOR FINAL

1 Compute the following limits.

$$\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\tan 3x}, \quad \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - x - 2}$$
$$\lim_{x \rightarrow \infty} \frac{3 - 2x + x^2 - 6x^3}{2x^3 + 5x - x^2 + \sqrt{3x^3}}, \quad \lim_{x \rightarrow -2^-} \frac{1 - 4x^2}{x^2 - 4}$$

2 Differentiate the following functions.

$$f(x) = \left(6x^5 - x^4 + 3\sqrt{2x} - \frac{3}{4x} + 101\right)^{17}, \quad g(x) = \sqrt{\sin^3 x + \cos(4x + 1)}$$
$$h(x) = \sin(\cos(\sin^2(3x - 5))), \quad s(x) = \frac{\sin 2x \sin 3x}{x + 7}$$

3 Find the following definite and indefinite integrals.

$$\int \sqrt{ax + b} \, dx \quad \int \sin^2(x^3 - 2)x^2 \cos(x^3 - 2) \, dx$$
$$\int_1^2 \frac{2x - 1}{(5x^2 - 5x)^3} \, dx \quad \int_0^1 \frac{2x^3 - 4x + 5\sqrt{x}}{\sqrt{x}} \, dx$$

4 Use implicit differentiation to solve for $\frac{dy}{dx}$ as a function of x and y for the equation

$$\sin(xy) - y^2 = 2x^3 + 1$$

5 (a) Let $f(x) = x^2$. Find c such that $f(c)$ is the average value of $f(x)$ on the interval $[0, 3]$.
(b) Let $f(x) = \frac{1}{3}x^3 + 5$. Find the c given by the Mean Value Theorem for derivatives on the interval $[0, 3]$.

6 Find the arc length of the curve parametrized by $x = \frac{1}{3}t^6 + 1$ and $y = \frac{1}{9}t^9 - 3$, for $0 \leq t \leq 1$.

7 Set up the integral that represents the surface area of the surface obtained by revolving the graph of $f(x) = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$ about the x -axis. **Do not solve the integral.**

8 Set up (do not solve) the integral that represents the volume of the solid generated by revolving the region bounded by $y = \frac{4}{x}$, $x = 1$, and $y = 1$ about the axes

(a) $x = 1$,

(b) $y = 0$.

9 Using the **shell method**, set up (do not solve) the integral that represents the volume of the solid generated by revolving the region bounded by $y = (x-2)^2 + 2$ and $y = 2(x-2)^2 + 1$ about the axis $x = 0$.

10 If it takes a force of 6 pounds to extend a spring 1 foot beyond its natural length. How much work does it take to compress the spring 3 feet?

11 Find the centroid (or center of mass) of the semicircle of radius 1. *Hint:* Use symmetry.

12 A small island is 2 miles from the nearest point P on the straight shoreline of a large lake. Her home is 10 miles down the shore from P . If a woman on the island can row a boat 3 miles per hour and can run 7 miles per hour, where should the boat be landed in order to arrive at her home in the least amount of time?

You aren't done. The graphing problem is in the next file, also downloadable on the course website.
