$$f(x) = 2(.01 - x^2)$$
$$y = 2(.01 - x^2)$$
$$x = 2(.01 - y^2) \qquad \text{Replacing } f(x) \text{ with } y$$
$$x = .02 - 2y^2 \qquad \text{Interchanging } y \text{ and } x\text{:}$$
$$2y^2 = .02 - x \qquad \text{Solving for } y$$
$$y^2 = \frac{.02 - x}{2}$$
$$y = \sqrt{\frac{.02 - x}{2}} \qquad \text{Only the positive value of } y \text{ is possible.}$$
$$f^{-1}(x) = \sqrt{\frac{.02 - x}{2}} \qquad \text{Replacing } y \text{ with } f^{-1}(x).$$

(c) The inverse function gives the distance from the center of the artery as a function of the velocity of a blood corpuscle.

2.6 Exercises

In Problems 1–4, using the functions

$$f(x) = \frac{1}{3x + 9},\ g(x) = 2x - 3,\ h(x) = \sqrt{x}$$

(a) Create the following functions

(b) Give the domain and range of the new functions.

1. $f(h(x))$
2. $g(h(x))$
3. $(f \circ g)(x)$
4. $(g \circ f)(x)$

In problems 5–8, find (a) $f(g(x))$ and (b) $g(f(x))$.

5. $f(x) = x^3;\ g(x) = 2x - 1$
6. $f(x) = 2x + 8;\ g(x) = \dfrac{1}{x^3}$
7. $f(x) = \sqrt{3x - 4};\ g(x) = \left(\dfrac{x}{3} + 4\right)$
8. $f(x) = \dfrac{4x - 2}{3};\ g(x) = \sqrt{4x - 1}$
9. Find two functions, f and g, such that their composition is

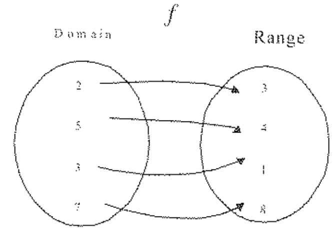
$$(f \circ g)(x) = (4x^3 + 5)^2$$

Find two functions, f and g, such that their composition is

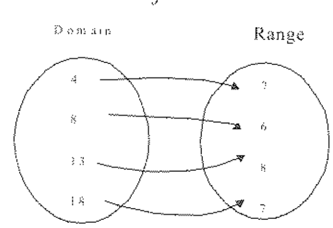
$$(f \circ g)(x) = \frac{1}{5x^3 + 4}$$

In each of Problems 11–12, determine if the function f defined by the arrow diagram has an inverse. If it does, create an arrow diagram that defines the inverse.

11. f



12. f



13. If $f(x) = 2x - 3$ and $g(x) = \dfrac{x + 3}{2}$:
    (a) What is $f(g(x))$?
    (b) Are $f(x)$ and $g(x)$ inverse functions?
14. If $f(x) = 5x - 1$ and $g(x) = \dfrac{x + 1}{5}$
    (a) What is $f(g(x))$?
    (b) Are $f(x)$ and $g(x)$ inverse functions?
15. If $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$, are $f(x)$ and $g(x)$ inverse functions?

16. If $f(x) = (x - 2)^3$ and $g(x) = \sqrt[3]{x + 2}$, are $f(x)$ and $g(x)$ inverse functions?

17. For the function f defined by $f(x) = 3x - 4$, complete the tables below for f and f^{-1}.

x	$f(x)$	x	$f^{-1}(x)$
-1		-7	
0		-4	
1	-7	-1	-1
2		2	
3		5	

18. (a) Write the inverse of $f(x) = 3x - 4$.

    (b) Do the values for f^{-1} in the table of Problem 17 fit the equation for f^{-1}?

19. If the graph of $y = f(x)$ has the point (a, b) on its graph, name a point on the graph of $y = f^{-1}(x)$.

20. Find the inverse of $f(x) = 6x + 5$

21. Find the inverse of $g(x) = \dfrac{1}{3x - 1}$.

22. $f(x) = 3x^2$ for $x \geq 0$.

23. Graph $g(x) = \sqrt{x}$ and its inverse $g^{-1}(x)$ for $x \geq 0$ on the same axes.

24. $f(x) = (x - 4)^2$ and $g(x) = \sqrt{x} + 4$ inverse functions for what values of x?

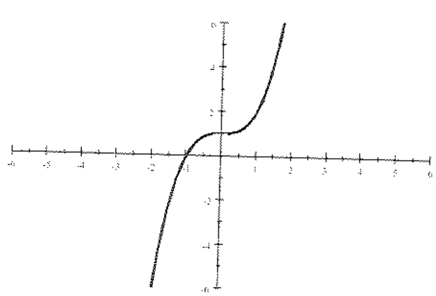
25. Is the function $f(x) = 5x^3 + 2$ a one-to-one function? Does it have an inverse?

26. Is the function $f(x) = 4x^2 + 1$ a one-to-one function? Does it have an inverse?

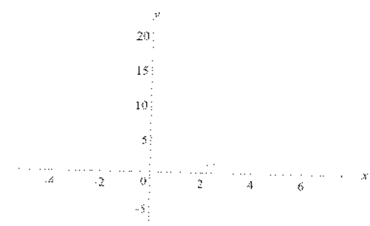
27. Is the function defined by $\{(1, 4), (2, 2), (3,3)\}$ a one-to-one function?

28. Is the function defined by $\{(1, 3), (6, 2), (4,3)\}$ a one-to-one function?

29. Sketch the graph of $y = f^{-1}(x)$ on the axes with the graph of $y = f(x)$, shown below.



30. Is the function with the graph below one-to-one?



APPLICATIONS

31. *Function composition* Think of each of the following processes as a function designated by the indicated letter. f: Placing in a Styrofoam container g: Grinding Describe each of the functions in (a) – (d), then answer part (f).

    (a) f(meat) (b) g(meat) (c) f(meat)) (d) g(meat))

    (f) Which of the functions in part c and part d gives a sensible operation?

32. *Function composition* Think of each of the following processes as a function designated by the indicated letter.

    f: putting a sock on g: taking a sock off

    Describe each of the functions in (a). – (c).

    (a) f(left foot) (b) f(left foot)) (c) $g \circ f$(right foot)

33. *Shoe sizes* A woman's shoe that is size x in Japan is size $s(x)$ in the United States, where $s(x) = x - 17$. A woman's shoe that is size x in the United States is size $p(x)$ in Britain, where $p(x) = x - 1.5$. Find a function that will convert Japanese shoe size to British shoe size. [*Source:* Kuru International Exchange Association]

34. *Shoe sizes* A man's shoe that is size x in Britain is size $d(x)$ in the United States, where $d(x) = x + 0.5$. A man's shoe that is size x in the United States is size $t(x)$ in Continental size, where $t(x) = x + 34.5$. Find a function that will convert British shoe size to Continental shoe size.

    [*Source:* Kuru International Exchange Association]

35. *Temperature and humidity* Let $t(x)$ be the temperature in degrees Celsius x hours after 12 noon and $h(t)$ be the relative humidity at temperature t degrees Celsius. Can t and h be combined by function composition? If so, write the notation for the new function and describe its input and output.

36. *Computer costs* Let $S(t)$ be the number of students attending your school t years after 2000 and $C(S)$ be the cost of computers (in dollars) purchased for labs when S students enroll in your school. Does the composite function $C \circ S$, the composite function $C \circ S$, neither of these, or both of these make sense? Give the output notation for and write a sentence giving the meaning of the function(s) that makes sense.

37. *Exchange rates* On January 19, 2001, each Austrian schilling was worth 1.987376 Russian rubles and each Chilean peso was worth 0.025202 Austrian schillings. Find the value of 1000 Chilean pesos in Russian rubles on January 19, 2001.
[*Source:* Expedia.com]

38. *Exchange rates* On January 19, 2001, each British pound sterling was worth 1.495701 US dollars and each French franc was worth 0.096115 British pound sterling. Find the value of 100 French francs in US dollars.

39. *Shoe sizes* A man's shoe that is size x in Britain is equivalent to size $d(x)$ in the United States, where

$$d(x) = x + 0.5.$$

(a) Find the inverse of the function.
(b) Use the inverse function to find the British size of a shoe if it is U.S. size $8\frac{1}{2}$.

[*Source:* Kuru International Exchange Association]

40. *Shoe sizes* A man's shoe that is size x in the United States is size $t(x)$ in Continental size, where

$$t(x) = x + 34.5.$$

(a) Find a function that will convert Continental shoe size to U.S. shoe size.
(b) Use the inverse function to find the U.S. size if the Continental size of a shoe is 43.

[*Source:* Kuru International Exchange Association]

41. *Cigarettes* For the years 1975 to 1991, the percent of high school seniors who have tried cigarettes is given by $f(t) = 75.451 - 0.707t$, where t is the number of years since 1975. Find the inverse of this function and use it to find the year in which the percent fell below 65%.
[*Source:* National Institute on Drug Abuse]

42. *Investments* If x is invested at 10% for 6 years, the future value of the investment is given by

$$S(x) = x + .6x.$$

(a) Find the inverse of this function.
(b) What can you find with this function?
(c) Use this function to find the amount of money that must be invested for 6 years at 10% to have a future value of 24,000.

43. *Ritalin* The function that models the consumption of Ritalin over the years 1990 to 1996 is

$$f(x) = 225.304x + 493.432$$

where x is the number of years from 1990 and y is measured in grams per 100,000 people.

(a) Find the inverse of this function. What can you find with this function?
(b) Use the inverse function to find when consumption equals at least 1000 grams per 100,000 people.
(c) Find and interpret $f(20)$.
(d) Find and interpret $f^{-1}(800)$.

44. *Apparent Temperature* If the outside temperature is 90°F, the Apparent Temperature is given by $A(x) = 82.35 + 29.3x$, where x is the humidity written as a decimal.

(a) Find the inverse of this function.
(b) Use the inverse to find the humidity that will give an Apparent Temperature of 97°.
(c) Find and interpret $f(0.65)$.
(d) Find and interpret $f^{-1}(100)$.

[*Source:* "Temperature-humidity Indices," The UMAP Journal, Fall 1989]

45. *Body-heat loss* The model for body-heat loss depends on the coefficient of convection $K = f(x)$, which depends on wind speed x according to the equation $f(x) = 4\sqrt{4x + 1}$.

(a) What is the domain and range of this function without regard to the context of this application?
(b) Find the inverse of this function.
(c) What are the domain and range of the inverse function?
(d) In the context of the application, what is the domain and range of the inverse function?

46. *Algorithmic relationship* For many species of fish, the weight W is a function of the length x, given by $W = kx^3$, where k is a constant depending on the species. Suppose that $k = .002$, W is in pounds, and x is in inches, so that the weight is $w(x) = 0.002x^3$

(a) Find the inverse function of this function.
(b) What does the inverse function give?
(c) Use the inverse function to find the length of a fish that weighs 2 pounds.
(d) In the context of the application, what is the domain and range of the inverse function?

47. *Decoding messages* If we assign numbers to the letters of the alphabet, and assign 27 to a blank space as is shown below, we can convert a message to a numerical sequence.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z	blank
14	15	16	17	18	19	20	21	22	23	24	25	26	27

We can "encode" a message by using the numbers to represent letters and we can further encode it by using the function $C(x)2x - 3$. Thus the message "Go for it" would be encoded as

11 27 51 9 27 33 51 15 37

Find the inverse of the function and use it to decode

37 13 7 51 33 7 -1 21 51 37 13 15 25 11.

48. ***Decoding messages*** Use the numerical representation from problem 47 and the inverse of the encoding function $C(x) = 3x + 2$ to decode

41 5 35 17 83 41 77 83 14 5 77.

49. ***Social Security numbers and income taxes*** Consider the function that assigns each person who pays federal income tax his or her social security number. Is this a 1-to-1 function? Explain.

50. ***Checkbook balance*** Consider the function with the check number in your checkbook as input and the dollar amount of the check as the output. Is this a 1-to-1 function? Explain.

51. ***Volume of a cube*** The volume of a cube is $f(x) = x^3$ cubic inches, where x is the length of the edge of the cube in inches.
   (a) Is this function one-to-one?
   (b) Find the inverse of this function.
   (c) What is the domain and range of this inverse function in the context of the application?
   (d) How could this function be used?

52. ***Volume of a sphere*** The volume of a sphere is $f(x) = \frac{4}{3}\text{p}x^3$ cubic inches, where x is the radius of the sphere in inches.
   (a) Is this function one-to-one?
   (b) Find the inverse of this function.
   (c) What is the domain and range of this inverse function?
   (d) How could this inverse function be used?
   (e) What is the radius of a sphere if its volume is 65,450 cubic inches?

53. ***Currency conversion*** The function that converts Canadian dollars to U.S. dollars according to the January 19, 2001 values is

$$f(x) = 0.66832x$$

where x is the number of Canadian dollars and $f(x)$ is the number of U.S. dollars.
   (a) Find the inverse function for f and interpret its meaning.
   (b) Use f and f^{-1} to determine the money you will have if you take 500 U.S. dollars to Canada, convert it to Canadian dollars, don't spend any, and then convert it back to U.S. dollars. (Assume there is no fee for conversion and the conversion rate remains the same.)

[*Source:* Expedia.com]

54. ***Surface area*** The surface area of a cube is $y = 6x^2$ square centimeters, where x is the length of the edge of the cube in centimeters
   (a) For what values of x does this model make sense? Is the model a one-to-one function for these values of x?
   (b) What is the inverse of this function on this interval?
   (c) How could the inverse function be used?

55. ***Illumination*** The intensity of illumination of a light is a function of the distance from the light. For a given light, the intensity is given by $I(x) = \frac{300{,}000}{x^2}$ candlepower, where x is the distance in feet from the light.
   (a) Is this function a one-to-one function?
   (b) What is the domain of this function in the context of the application?
   (c) Is the function one-to-one for the domain in (b)?
   (d) Find the inverse of this function and use it to find the distance at which the intensity of the light is 75,000 candlepower.

56. ***Supply*** The supply function for a product is $p(x) = \frac{1}{4}x^2 + 20$, where x is the number of thousands of units a manufacturer will supply if the price is $p(x)$.
   (a) Is this function a one-to-one function?
   (b) What is the domain of this function in the context of the application?
   (c) Is the function one-to-one for this domain?
   (d) Find the inverse of this function and use it to find how many units the manufacturer is willing to supply if the price is \$101.

57. ***Exchange rates*** Suppose that the function that converts United Kingdom (U.K.) pounds to U.S. dollars is

$$f(x) = 1.7655x$$

where x is the number of pounds and $f(x)$ is the number of U.S. dollars.
   (a) Find the inverse function for f and interpret its meaning.
   (b) Use f and f^{-1} to determine the money you will have if you take \$1000 U.S. to the U.K., convert it to pounds, don't spend any, and then convert it back to U.S. dollars. (Assume there is no fee for conversion and the conversion rate remains the same.)

[*Source:* International Monetary Fund]

58. ***First class postage*** The postage charged for first-class mail is a function of its weight. The U.S. Postal Service uses the following table to describe the rates.

Weight increment, w	Postal Rate
First ounce or fraction of an ounce	\$0.410
Each Additional ounce	\$0.230 more

[*Source:* pe.usps.gov/text]

(a) Convert this table to a piecewise-defined function $P(x)$ that represents postage for letters weighing more than 0 and no more than 3 ounces, using w as the weight in ounces and $P(x)$ the postage in cents.

(b) Does P have an inverse function? Why or why not?