Practice Exam 3 – MATH 1170, Fall 2007

1. Compute the first and second derivatives of the following functions:
(a) $f(x) = x^6$
(b) $f(x) = x^{-3.5}$
(c) $f(x) = x - \frac{x^3}{6} + \frac{x^5}{120}$ (d) $g(t) = (2t^3 + t)(4t^2 + 1)$
(e) $h(t) = \frac{(t+1)(2t+3)}{3t+2}$
(f) $F(x) = x^3 \mathrm{e}^{-x}$
(g) $G(x) = \frac{\ln(2x)}{x}$
(h) $P(x) = \mathrm{e}^{-x^4}$
(i) $L(x) = \ln(2x^2)$
(j) $M(x) = \ln(\mathrm{e}^x)$

2. If Bernice, the duck-billed platypus, lays N eggs, the probability of an egg successfully producing an offspring is $P(N) = \frac{0.1N+0.8}{(1+N)^2}$.
(a) Calculate the predicted surviving offspring $S(N)$ when Eunice lays 1, 5, and 10 eggs.
(b) Calculate the derivative of $S(N)$.
(c) Draw a graph of the function $S(N)$.
(d) Does the derivative $S'(N)$ ever equal zero?
(e) What is Bernice best strategy if she wants to maximize her offspring output?

3. Elmo throws a ball from the top of a tower on the surface of the asteroid Eris (aka Xena). The ball's distance above the ground follows the equation $p(t) = -0.8t^2 - t + 200$.
(a) What is the velocity of the ball?
(b) What is the acceleration of the ball?
(c) How high was the tower from which Elmo threw the ball? What direction did he throw it?

4. The quantity of truffles on a farm in the south of France as a function of time follows the equation $Q(t) = 100\mathrm{e}^t$ where t is in months. Each individual truffle's mass follows the equation $M(t) = 1 - \frac{t}{10}$.
(a) Calculate the total mass of truffles as a function $T(t)$ of time.
(b) What is $T(0)$?
(c) For what value of t does $T(t)$ equal zero?
(d) Calculate the derivative of $T(t)$.
(e) What is $T'(0)'$? Is total mass increasing or decreasing at $t = 0$?
(f) For what value of t does the derivative of $T(t)$ equal zero?
(g) Graph $T(t)$.
(h) Truffles are very valuable. If you wanted to maximize the total mass of a truffle harvest, at what time should you perform this harvest?

5. Define $F(x) = 1 - \mathrm{e}^{-x^2}$, what is the derivative of the inverse $F^{-1}(x)$?

6. For the discrete-time dynamical system $a_{t+1} = 3a_t - 3$:
(a) Plot the updating function and the diagonal over the domain $0 \leq a_t \leq 5$, and cobweb for four steps, starting at $a_0 = 2$.
(b) Calculate the equilibrium of this system algebraically.
(c) Find the derivative of the updating function $f(a)$.
(d) Use the derivative of the updating function to determine the stability of the equilibrium. Does this match your finding in part (a)?

7. The per capita production of a population as a function of current population x is $\frac{5x}{6+x^2}$.
(a) Write down the discrete time dynamical system for how population changes with each timestep.
(b) Calculate all of the equilibria algebraically.
(c) Draw the updating function and the diagonal on the same axis and cobweb starting at a few different places.
(d) Calculate the derivative of the updating function.
(e) Find the stability of all of the equilibria using the derivative of the updating function.
(f) Comment on these findings.

8. Bubonic plague viral load (in thousands) is given as a function of time (in months) $B(t) = t^6 - 12t^2 + 40$, for the domain $0 \leq t \leq 10$ months.
(a) Calculate the value of $B(t)$ at the endpoints $t = 0$ and $t = 10$.
(b) Find any critical points in the domain. What is the viral load at these points?
(c) The plague can be treated with Gentamicin, but works most effectively when administered at a time when the viral load is at a minimum. What is the global minimum of $B(t)$ on the domain $0 \leq t \leq 10$ months? At what time does this occur?