Practice Exam 1 Problems – MATH 1170, Fall 2007

Show all your work. Simplify as much as possible.

1. The $CD4^+$ T Cells of an HIV patient follow the discrete-time dynamical system $c_{t+1} = 0.5c_t$, $c_0 = 1 \times 10^7$, where t is in years.
(a) What is the solution to this discrete-time dynamical system?
(b) Graph the solution.
(c) What is the half-life of the T cell population?
(d) How long will it take for the patient's T cells to reach the level $c_t = 1 \times 10^5$?

2. In a fetus, the number of neuronal synapses in the brain follows the discrete-time dynamical system $s_{t+1} = 1.8s_t$, $s_0 = 1 \times 10^3$, where t is in weeks.
(a) What is the solution to this discrete-time dynamical system?
(b) Graph the solution.
(c) What is the doubling time of the synapses?
(d) How long will it take for the fetus' synapse number to reach the level $s_t = 1 \times 10^{10}$?

3. For $f(t) = 2t^2 + 1$:
(a) Find the average rate of change between times $t_0 = 0$ and $t_0 + \Delta t = \Delta t$ for $\Delta t = 1.0, 0.5, 0.1, 0.01$.
(b) Using the values from (a), guess what the instantaneous rate of change is at $t_0 = 0$.
(c) Draw a graph of the function $f(t)$, and the tangent line at the point $(0, f(0))$.

4. For $f(t) = e^{3t}$:
(a) Find the average rate of change between times $t_0 = 0$ and $t_0 + \Delta t = \Delta t$ for $\Delta t = 1.0, 0.5, 0.1, 0.01$.
(b) Using the values from (a), guess what the instantaneous rate of change is at $t_0 = 0$.
(c) Draw a graph of the function $f(t)$, and the tangent line at the point $(0, f(0))$.

5. Use the following tables of functions $f(x)$ at given values of x

x	$f(x) = (1+x)^{1/x}$
0.1	2.594
10^{-5}	2.718
10^{-10}	2.718

x	$f(x) = \frac{1-\cos(x)}{x}$
0.1	1.52×10^{-5}
0.01	1.52×10^{-6}
0.001	1.52×10^{-7}

to find
(a)

$$\text{(a)} \quad \lim_{x \to 0^+} (1+x)^{1/x} \qquad \text{(b)} \quad \lim_{x \to 0^+} \frac{1-\cos(x)}{x}$$

(c)

$$\lim_{x\to 0^+} \frac{(1+x)^{1/x}x}{1-\cos(x)}$$

6. For $f(x) = x^4$, how close must x be to 0 for
(a) $f(x)$ to be within 0.1 of 0?
(b) $f(x)$ to be within 0.0001 of 0?
(c) Sketch a graph of the function and say if $f(x)$ approaches 0 quickly or slowly and why.

6. For $f(x) = \frac{1}{x^3}$, how close must x be to 0 for
(a) $f(x)$ to be greater than 10?
(b) $f(x)$ to be greater than 1000?
(c) Sketch a graph of the function and say if $f(x)$ approaches infinity quickly or slowly and why.

7. Find the average rate of change for $f(x) = 2x^2 + 3x + 1$ near $x = 1$ as a function of Δx and find the limit as $\Delta x \to 0$. Graph the function, and indicate the tangent to the point $(1, f(1))$.

8. Are the following functions continuous or discontinuous? Indicate points of discontinuity.
(a) $f(x) = (1-x)^{-4}$
(b) $f(x) = \sin(e^{x^2})$
(c) $f(x) = \frac{e^x}{x}$
(d) $f(x) = (1+x^2)^{-2}$

9. Find
(a)

$$\lim_{x\to 0^+} \frac{1}{x}$$

(b)

$$\lim_{x\to 0^-} \frac{1}{x}$$

10. An object dropped from a height of 100m has distance above the ground

$$M(t) \quad = \quad 100 - \frac{a}{2}t^2$$

where a is the acceleration of gravity. If the object is on Jupiter, where $a = 22.88\text{m/s}^2$, find the time when the object hits the ground and its speed at that time.