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Research Statement
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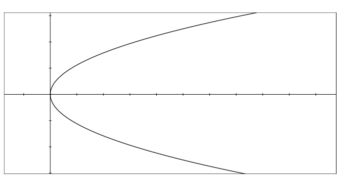
1. Introduction

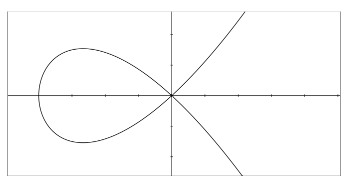
My research concerns topics in the areas of complex algebraic geometry and positive characteristic commutative algebra. I specialize in multiplier ideals and singularities on complex algebraic varieties and their mysterious correspondence to certain algebraic properties of the Frobenius map for commutative rings containing a finite field. In the coming years, I plan to advance these subjects and the connections between them.

Algebraic geometry is one of the oldest and yet most active disciplines in mathematics. The field has strong ties to such diverse areas as complex analysis, topology, and number theory, and it is used in a wide variety of applied settings. Applications range from error-correcting codes in computer science and genomics to control theory and modeling in engineering.

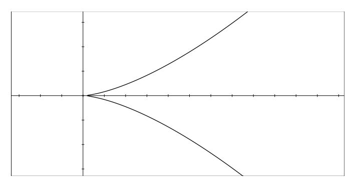
Algebraic geometers seek to understand algebraic varieties, geometric objects given locally as the solutions to polynomial equations. For instance, a plane curve is the zero set of a polynomial in two variables (see below for some examples). The richness and simplicity of polynomial equations make algebraic varieties fascinating objects of study.



Parabola $y^2 = x$



Node $y^2 = x^3 + x^2$



Cusp $y^2 = x^3$

The singular (or non-manifold) points of a complex algebraic variety have subtle local structure, and detailing their properties – even in the study of smooth varieties – is a critical part of many investigations. For example, the seminal work [BCHM] proves the existence of a distinguished birational modification or canonical model for every smooth complex projective variety (*cf.* [Siu06]). This model is produced via the so-called Minimal Model Program, wherein it is essential to control the singularities appearing in steps along the way. These include log terminal, log canonical, rational, and Du Bois singularities [Rei87, KMM87, K+92, KM98], and their behavior is often understood through the use of multiplier ideals and related invariants such as log canonical thresholds and jumping numbers [Ein97, Laz04].

Constructing and assigning invariants to singularities allows us to compare them with one another. For instance, the degree of the smallest order term in the equation of a plane curve is called its multiplicity. A smooth curve such as the parabola $y^2 = x$ has multiplicity one, while both the node $y^2 = x^3 + x^2$ and the cusp $y^2 = x^3$ have multiplicity two. More subtle invariants of singularities are defined using a resolution or smoothing. For example, to resolve the node $y^2 = x^3 + x^2$, one can imagine lifting a loop of string off of a table so that it no longer overlaps itself; the cusp $y^2 = x^3$ has a more complicated resolution. Resolving singularities can be very difficult in general, and recently described invariants called jumping numbers give some measure of the complexity of this process [ELSV04]. In particular, jumping numbers give a precise sense in which the singularity of the cusp is worse than the singularity of the node; the smallest jumping number of the node $y^2 = x^3 + x^2$ is 1, while for the cusp $y^2 = x^3$ it is $\frac{5}{6}$.

My doctoral thesis and the resulting publications make extensive progress in understanding multiplier ideals and jumping numbers on complex algebraic surfaces. In the first paper [Tuc08a], I answer a question posed in [LLS08] by showing that every complete ideal on a log terminal surface is a multiplier ideal. The second paper [Tuc08b] gives an explicit algorithm for computing the jumping numbers of an arbitrary curve on a smooth surface from the numerical data of its minimal resolution. Furthermore, in higher dimensions, Karen Smith and I have made progress towards identifying the essential valuations for the computation of multiplier ideals.

In a completely different direction, commutative algebraists have long used the Frobenius or p-th power map to study commutative rings containing a finite field. The theory of tight closure and test ideals, for example, has widespread applications such as the verification of difficult homological conjectures for equi-characteristic local rings [HH92, Hun96]. Furthermore, the algebraic analogue of a smooth variety is a regular ring (such as a polynomial ring over a field), and the failure of rings to be regular can be detected using Frobenius. As a result, tight closure leads to definitions of F-regular, F-rational, F-pure, and F-injective singularities [HR76, Fed83, HH90, Smi01].

Surprisingly, multiplier ideals and singularities on complex algebraic varieties are closely related to the classifications and constructions arising independently in the theory of tight closure. Given a complex algebraic variety, standard reduction to characteristic p techniques produce a family of positive characteristic models. When the variety is defined by polynomial equations with integer coefficients (such as the examples of plane curves given above), these are simply defined by reducing the equations modulo each of the prime numbers. This opens the door to the use of powerful Frobenius techniques in characteristic zero: a complex algebraic variety has F-regular, F-pure, F-rational, or F-injective type if a Zariski-dense set of its positive characteristic models has the relevant property. In this manner, log terminal and F-regular singularities, log canonical and F-pure singularities, rational and F-rational singularities, and Du Bois and F-injective singularities are either known or conjectured to correspond to one another [Smi97, Har98a, MS97, HW02, Sch09b]. Furthermore, the reduction of the multiplier ideal coincides with the test ideal for all but finitely many positive characteristic models [Smi00, HY03, Har01, Tak04].

This link is a great source of intuition, and has led to interesting results in both algebraic geometry and commutative algebra. Recently, Karl Schwede and I have described the behavior of test ideals under generically separable finite morphisms in analogy to multiplier ideals [ST09a]. In a separate project, we also find effective sharp bounds for the number of either log canonical centers or compatibly Frobenius split subvarieties [ST09b]. I believe the interplay of geometric methods in characteristic zero and Frobenius techniques in characteristic p will continue to inspire new questions and results which test and improve our knowledge of the singularities arising in both of these fields. My proposed research continues to exploit this connection.

2. Research Objectives, Methods, and Significance

Below are three specific topics and directions I am planning to pursue in my research.

(2.1) Transformations of Test Ideals under Generically Finite Morphisms.
(2.2) Multiplier Ideals and Test Ideals without $\mathbb{Q}$-Gorenstein Assumptions.
(2.3) Essential Valuations for Computing Multiplier Ideals and Jumping Numbers.

The first and third are natural extensions of my previous work, and the second is motivated by the work of de Fernex and Hacon [DH09]. Concrete goals, previous results, and possible approaches are detailed in the following sections. Even partial answers to the proposed questions would be of interest to a broad mathematical audience.

2.1. **Transformations of Test Ideals under Generically Finite Morphisms.** Pairs (X, Δ) where X is a normal algebraic variety and Δ an effective $\mathbb{Q}$-divisor arise naturally throughout complex algebraic geometry. For example, these divisors may represent the boundaries of open varieties, markings in moduli problems, or simply error terms in adjunction formulae. Traditionally, one considers only pairs (X, Δ) which are log $\mathbb{Q}$-Gorenstein, *i.e.* such that one can find an integer m where $m(K_X + \Delta)$ is a Cartier divisor. The multiplier ideal $\mathcal{J}(X, \Delta)$ is a crucial tool which measures the singularities of such pairs; worse singularities correspond to deeper multiplier ideals [BL04]. Many fundamental properties of multiplier ideals stem from their computability via a log resolution $\pi : Y \to X$. Precisely, if (X, Δ) is log $\mathbb{Q}$-Gorenstein, then

$$\mathcal{J}(X, \Delta) = \pi_* \mathcal{O}_Y(\lceil K_Y - \pi^*(K_X + \Delta) \rceil). \tag{†}$$

More generally, if $f : Z \to X$ is a generically finite proper morphism of normal complex varieties and (X, Δ) is a log $\mathbb{Q}$-Gorenstein pair, then

$$\mathcal{J}(X, \Delta) = f_* \left(\mathcal{J}(Z, f^*(K_X + \Delta) - K_Z) \right) \cap \mathbb{C}(X). \tag{1}$$

where the right hand side must be interpreted as the multiplier ideal of a possibly non-effective divisor [Laz04].

Under the reduction process described in the introduction, the multiplier ideals $\mathcal{J}(X, \Delta)$ of log $\mathbb{Q}$-Gorenstein pairs (X, Δ) correspond to (generalized) test ideals $\tau(X, \Delta)$ from the theory of tight closure. Test ideals have an intricate definition via Frobenius, and properties of multiplier ideals which are immediate from (†) are often difficult to prove for test ideals. Conversely, deep theorems about multiplier ideals requiring subtle vanishing theorems are often very easy to show for test ideals. Nevertheless, for a generically separable morphism, it is reasonable to expect a formula analogous to (1):

Question 1. How do test ideals transform under proper morphisms which are both generically finite and separable?

A solution to this problem would have many interesting applications, such as computability statements for test ideals via alterations [dJ96]. In addition, one might be able to describe how the singularities of tight closure theory behave under the steps in a minimal model program in positive characteristic. Showing the existence of such a program is an area of active research.

The technique I propose to use in addressing Question 1 stems from the correspondence in (2) below. If $X = \mathrm{Spec}(R)$ for a normal F-finite local domain of characteristic $p > 0$, there is a bijection [HW02, Sch09a]:

$$\left\{ \begin{matrix} \text{Effective } \mathbb{Q}\text{-divisors } \Delta \text{ on } X \text{ such} \\ \text{that } (p^e - 1)(K_X + \Delta) \text{ is Cartier} \end{matrix} \right\} \longleftrightarrow \left\{ \begin{matrix} \text{Maps } \phi : F^e_* \mathcal{O}_X \to \mathcal{O}_X \text{ up to premulti-} \\ \text{plication by an element of } H^0(X, \mathcal{O}_X)^* \end{matrix} \right\}. \tag{2}$$

Note that each of the pairs (X, Δ) appearing on the left are automatically log $\mathbb{Q}$-Gorenstein. This equivalence follows from a relatively straightforward duality argument appearing as early as [MR85, Fed83], and it allows one to translate geometric statements involving pairs into purely algebraic statements about maps of coherent sheaves. By analyzing the lifting properties of these maps, I hope to find a satisfactory answer to Question 1. I have already used this approach to address the case of a finite morphism in very recent joint work with Karl Schwede [ST09a]; the trace map, which encodes subtle arithmetic information, plays a central role in our analysis.

Theorem 2 ([ST09a])**.** *Suppose $f : Z \to X$ is a generically separable finite morphism of normal varieties over a perfect field K of positive characteristic. Let $Tr \colon f_* \mathcal{O}_Z \to \mathcal{O}_X$ be the $\mathcal{O}_X$-module homomorphism corresponding to the trace map of the extension of function fields $K(X) \subseteq K(Z)$. Then for any log $\mathbb{Q}$-Gorenstein pair (X, Δ), we have*

$$\tau(X, \Delta) = Tr \left(\tau \left(Z, f^*(K_X + \Delta) - K_Z \right) \right).$$

Additionally, if $Tr : f_* \mathcal{O}_Z \to \mathcal{O}_X$ is surjective, then

$$\tau(X, \Delta) = \tau(Z, f^*(K_X + \Delta) - K_Z) \cap K(X).$$

As in the statement for multiplier ideals, $\tau(Z, f^*(K_X + \Delta) - K_Z)$ is the test ideal corresponding to a possibly non-effective divisor. In light of Theorem 2, Question 1 reduces to understanding how test ideals transform under proper birational morphisms, such as resolutions.

The correspondence in (2) is an important source of motivation and intuition. For example, consider the following Theorem, which I have recently proved in (distinct) joint work with Karl Schwede:

Theorem 3 ([ST09b]). *Suppose X is an affine algebraic variety over a field k, and $x \in X$ is a point of embedding dimension n.*

- *(i) If $k = \mathbb{C}$ and Δ is a boundary divisor such that (X, Δ) has log canonical singularities, then there are at most $\binom{n}{d}$ log canonical centers of (X, Δ) with dimension d passing through x.*
- *(ii) If $\mathrm{char}(k) = p > 0$ and $\phi : F_*\mathcal{O}_X \to \mathcal{O}_X$ is an F-splitting, then there are at most $\binom{n}{d}$ subvarieties of X with dimension d passing through x which are ϕ-compatibly F-split.*

After reduction to positive characteristic, the choice of boundary divisor Δ in (i) is essentially equivalent to the choice of F-splitting ϕ in (ii). Further analysis – also using (2) – relates the log canonical centers of (X, Δ) and ϕ-compatibly F-split subvarieties [Sch08]. Thus, it is reasonable to expect these very different objects to satisfy the same (sharp) bounds. Given the ubiquity of splittings of Frobenius in tight closure theory, one would expect the use of pairs in positive characteristic commutative algebra to continue to grow in importance in the coming years.

2.2. **Multiplier Ideals and Test Ideals without log $\mathbb{Q}$-Gorenstein Assumptions.** In positive characteristic, the definition of the test ideal $\tau(X, \Delta)$ naturally extends to pairs (X, Δ) which are not log $\mathbb{Q}$-Gorenstein. Recently, de Fernex and Hacon [DH09] have similarly extended multiplier ideals $\mathcal{J}(X, \Delta)$ to arbitrary pairs (X, Δ) by using valuation theoretic methods to approximate pull-backs of Weil divisors. Alternatively, $\mathcal{J}(X, \Delta)$ can be defined in terms of the multiplier ideals of larger log $\mathbb{Q}$-Gorenstein pairs:

Theorem 4 ([DH09], Corollary 5.5). *If X is normal complex variety and Δ is an effective $\mathbb{Q}$-divisor, then $\mathcal{J}(X, \Delta)$ is the unique maximal element of*

$$\{\mathcal{J}(X, \Delta + \Xi) : \Xi \text{ is an effective } \mathbb{Q}\text{-divisor and } (X, \Delta + \Xi) \text{ is log } \mathbb{Q}\text{-Gorenstein}\}.$$

The extension of multiplier ideals to this setting raises a number of interesting and important questions. Perhaps the most important is the following:

Question 5 ([DH09], page 3). Does the multiplier ideal of a complex pair which is not log $\mathbb{Q}$-Gorenstein correspond to the test ideal under reduction to positive characteristic?

There is evidence to suggest a positive answer to this question; Blickle [Bli04] has verified the case of toric varieties and Schwede [Sch09c] has proven an analogue of Theorem 4 for test ideals.

Other interesting questions concern the numerical invariants associated to arbitrary pairs. If X is a normal variety and D is an effective $\mathbb{Q}$-divisor on X, it is natural to consider the family of pairs $(X, \lambda D)$ for all $\lambda \in \mathbb{Q}_{>0}$. When X has characteristic zero, increasing λ can only result in a smaller multiplier ideal. A jumping number $\xi \in \mathbb{R}_{>0}$ of (X, D) has the property that $\mathcal{J}(X, \lambda_2 D) \subsetneq \mathcal{J}(X, \lambda_1 D)$ for all positive rational numbers λ_1 and λ_2 with $\lambda_1 < \xi < \lambda_2$ [ELSV04]. If X is log terminal, the smallest jumping number is also known as the log canonical threshold. Similarly, when X has positive characteristic, $\xi \in \mathbb{R}_{>0}$ is said to be an F-jumping number of (X, D) if $\tau(X, \lambda_2 D) \subsetneq \tau(X, \lambda_1 D)$ for all positive rational numbers λ_1 and λ_2 with $\lambda_1 < \xi < \lambda_2$ [MTW05]. If X is F-regular, the smallest F-jumping number is known as the F-pure threshold of (X, D).

When X is $\mathbb{Q}$-Gorenstein and D is $\mathbb{Q}$-Cartier, all of the pairs $(X, \lambda D)$ are log $\mathbb{Q}$-Gorenstein. In this case in characteristic zero, it is immediate from (†) that the jumping numbers of (X, D) are a discrete set of rational numbers. In the general case, however, while (†) holds for a log $\mathbb{Q}$-Gorenstein pair $(X, \lambda D + \Xi)$ exhibiting $\mathcal{J}(X, \lambda D + \Xi) = \mathcal{J}(X, \lambda D)$ as in Theorem 4, the choice of Ξ depends on λ. As a result, one cannot compute all of the ideals $\mathcal{J}(X, \lambda D)$ for $\lambda \in \mathbb{Q}_{>0}$ from a single log resolution, and discreteness and rationality of the jumping numbers is unclear.

Question 6 (*cf.* [DH09], Remark 4.10)**.** Suppose X is a normal complex algebraic variety which is not $\mathbb{Q}$-Gorenstein and D is any effective $\mathbb{Q}$-divisor. If X has characteristic zero, are the jumping numbers of (X, D) a discrete and rational? Similarly, if X has positive characteristic, are the F-jumping numbers of (X, D) discrete and rational?

It is reasonable to expect compatible answers in Question 6. Note that, when X is $\mathbb{Q}$-Gorenstein and D is $\mathbb{Q}$-Cartier in positive characteristic, it is a recent and non-trivial result [BSTZ09] that the F-jumping numbers are discrete and rational.

As a first line of attack in characteristic zero towards addressing the questions above, I plan to closely examine multiplier ideals for arbitrary pairs (X, Δ) when X is a complex surface. In this situation, the numerical pullback $(\pi^*)_{\text{num}}(F)$ of any divisor F under a resolution $\pi : Y \to X$ is well-defined (see [KM98], Section 4.1) and preceeds [DH09]. In fact, I have previously made use of this operation in [Tuc08a], where a proof of the following Theorem was given without using the non-trivial fact that every divisor on a (numerically) log terminal surface is automatically $\mathbb{Q}$-Cartier. This result provides an answer to a question of Lazarsfed, Lee, and Smith posed in [LLS08], and extends the analogous result for smooth surfaces shown in [FJ05, LW03].

Theorem 7 ([Tuc08a])**.** *If X is a (numerically) log terminal complex surface, then every integrally closed ideal is a multiplier ideal.*

Using numerical pullback, one can define a numerical multiplier ideal $\mathcal{J}_{\text{num}}(X, \Delta)$ for arbitrary pairs and compare it to the multiplier ideal $\mathcal{J}(X, \lambda D)$ defined in [DH09].

Question 8. If X is a normal complex surface and the pair (X, Δ) is arbitrary, what is the relationship between $\mathcal{J}(X, \Delta)$ and $\mathcal{J}_{\text{num}}(X, \Delta)$?

One strategy would be to begin by computing specific examples where X is the cone over smooth projective curve C of genus ≥ 2 normally embedded via an ample divisor which is not Q-linearly equivalent to a rational multiple of K_C.[1] I hope to simultaneously pursue an alternative in positive characteristic by addressing the following classification problem of independent interest.

Problem 9. Classify all F-regular and F-pure two-dimensional pairs (X, Δ).

One can reasonably expect to find a solution as the classification has already been given in the case $\Delta = 0$ by Hara in [Har98b], building on [MS91, Sri91]. Additionally, the corresponding classification in characteristic zero is complete [K+92, KM98].

2.3. **Essential Valuations for Computing Multiplier Ideals and Jumping Numbers.** If D is a $\mathbb{Q}$-Cartier divisor on a $\mathbb{Q}$-Gorenstein normal complex variety X and $\lambda \in \mathbb{Q}_{>0}$, it can be useful when thinking about $\mathcal{J}(X, \lambda D)$ to translate (†) into the language of valuations. On an affine open subset $U \subset X$, we have that $f \in H^0(U, \mathcal{O}_X)$ is an element of the multiplier ideal $\mathcal{J}(X, \lambda D)$ if and only if the conditions

$$\text{ord}_E(f \circ \pi) \geq \text{ord}_E(\lfloor \pi^*(K_X + \lambda D) \rfloor) \tag{3}$$

are satisfied for all prime divisors E on Y over U, where ord_E is the divisorial valuation corresponding to E. From this perspective, it is natural to ask if there is a minimal or best set of valuations determining membership in $\mathcal{J}(X, \lambda D)$.

Question 10. Which valuations are essential in the computation of the multiplier ideals $\mathcal{J}(X, \lambda D)$? Can we view identify them as the valuations corresponding to a particular birational model of X?

[1] It has been brought to my attention that de Fernex, Boucksom, and Favre may have another approach to Question 8 using certain functions on spaces of valuations.

An analagous question arises when computing the integral closures of ideals in commutative rings, where the essential valuations are known as the Rees valuations. These are precisely the valuations appearing on the normalized blow-up of the ideal [HS06]. Since multiplier ideals can be computed from any log resolution, one would expect an essential valuation to come from a divisor appearing on every log resolution. An answer to the above question would significantly improve our ability to compute multiplier ideals and jumping numbers and reveal more of the algebraic and geometric data which they encode.

Smith and Thompson [ST07] explored this question for plane curves and introduced the notion of jumping number contribution by a prime divisor. I extended this concept in [Tuc08b], and Question 10 can be reformulated in this language as: which divisors (critically) contribute to the jumping numbers of (X, D)? Together, [ST07] and [Tuc08b] give a complete answer when D is a curve on a smooth surface X, and I applied these results to show the following Theorem.

Theorem 11 ([Tuc08b])**.** *Let X be a normal complex surface with rational singularities. For any effective $\mathbb{Q}$-divisor D on X, there is an algorithm for computing the jumping numbers of (X, D) using only the numerical data of its minimal log resolution.*

The algorithm presented is both simple and constructive; see [Tuc08b, Section 6] for further details. This result has many interesting applications, including an alternate proof of the formula for the jumping numbers of an analytically irreducible plane curve given in [Jär06] (*cf.* [Nai09]). Furthermore, these methods yield examples of non-equisingular plane curves whose jumping numbers coincide.

Although many of the above techniques are specific to dimension two, it is likely that ideas from the Minimal Model Program can be used to address Question 10 in higher dimensions. For instance, Karen Smith and I have shown the following (unpublished) result:

Theorem 12. *Let X be a smooth complex algebraic variety, D an effective divisor on X, and $\pi : Y \to X$ a log resolution of D. Then the relative log canonical model of $(Y, (\pi^* D)_{\mathrm{red}})$ is independent of the chosen resolution and can be used to compute $\mathcal{J}(X, \lambda D)$ for all $\lambda \in \mathbb{Q}_{>0}$.*

In particular, the essential divisors are a subset of the divisors living on the relative log canonical model appearing in Theorem 12. Hopefully, further analysis will identify which of the divisors on the log canonical model are essential for the computation of multiplier ideals.

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