A Medley of Mathematical Models

to compliment the families of functions

Presented Saturday, October 13, 2001 at the UCTM conference at Brighton High School

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A model for each family in the families of functions to excite your students and help them relate math to the real world.

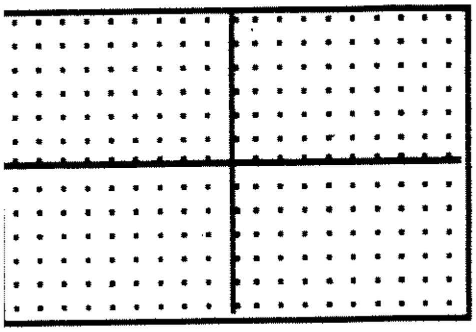
Linear:	Car Depreciation or other linear data selected by students
Quadratic:	Height as a function of time for an object thrown straight up
Cubic:	Volume of a box folded from a sheet of paper.
Rational:	Known volume exercise with cylinders
Radical:	The period of the pendulum as a function of the length
Exponential:	Inflation project
Logarithmic:	Modem price as a function of speed.
Sinusoidal:	Temperature or length of day data
Sequences:	Height of ball as a function of the number of the bounce.

QUICK-DRAW, MCGRAW!

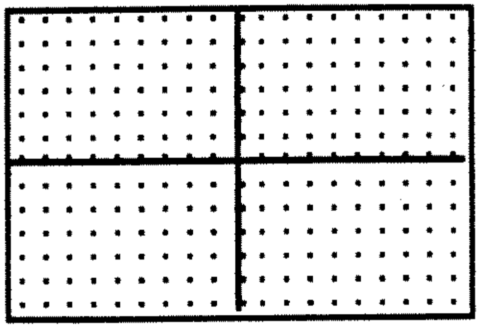
Name______________________ Pd._____

Equation type____________________

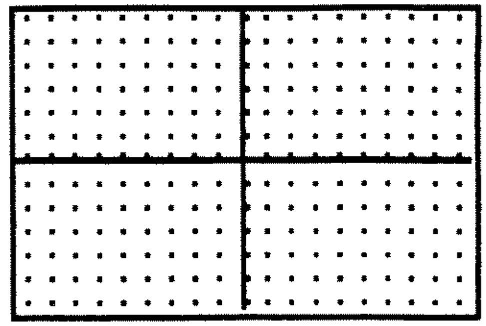
1. __________________



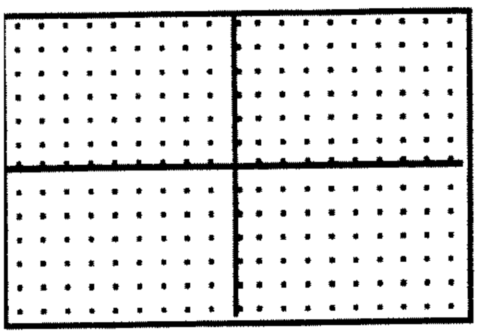
2. __________________



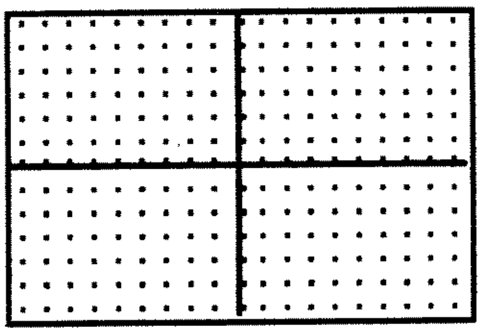
3. __________________



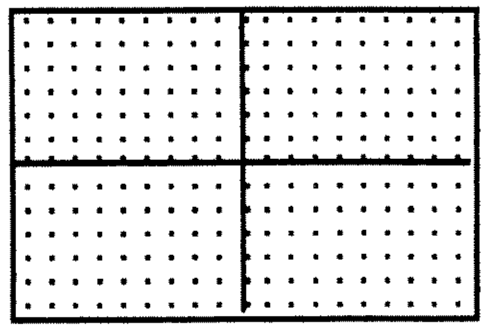
4. __________________



5. __________________



6. __________________

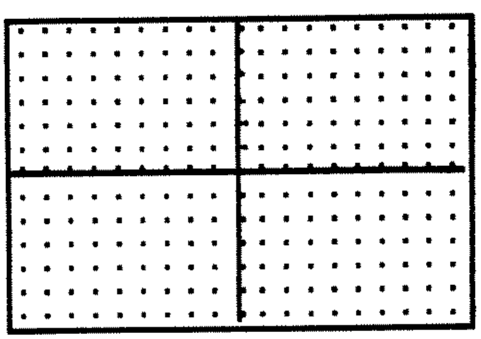


QUICK-DRAW, MCGRAW!

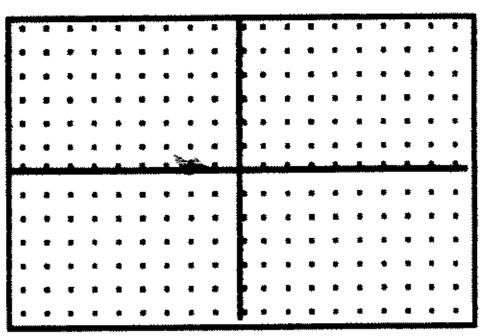
Name______________________ Pd._____

Equation type____________________

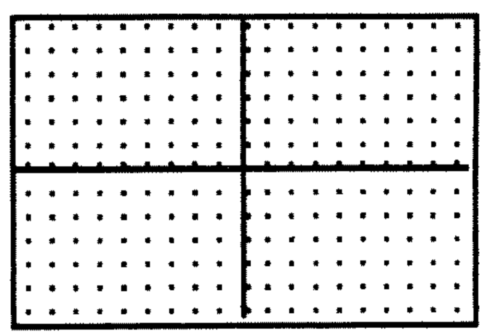
1. __________________



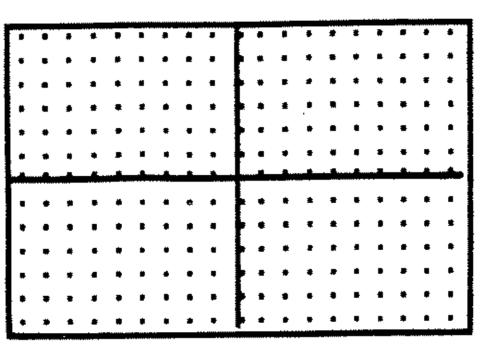
2. __________________



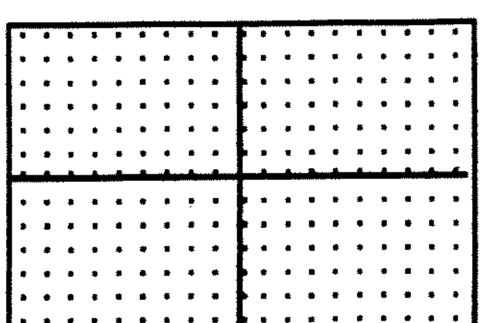
3. __________________



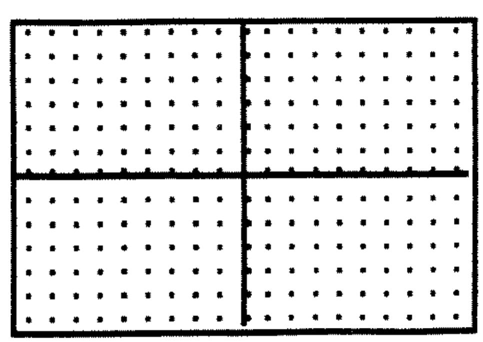
4. __________________



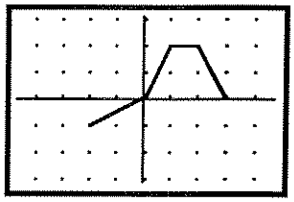
5. __________________



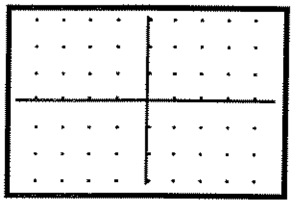
6. __________________



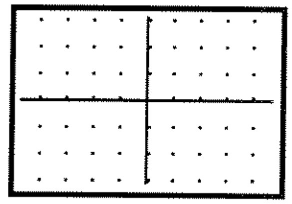
Shifty the function and the great STretch, REflect, Shift, Shift and the absolute value of it all!



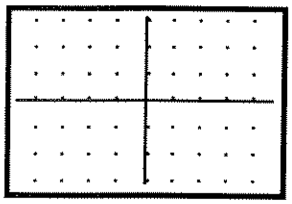
f(x)



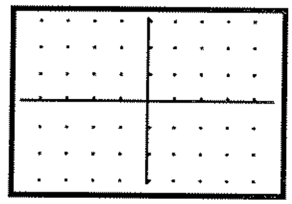
2f(x)



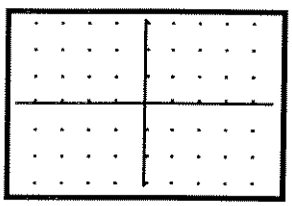
f(2x)



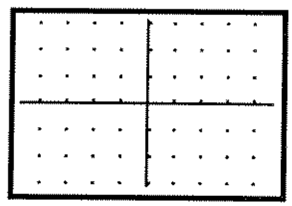
- f(x)



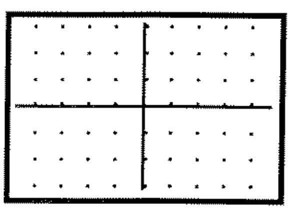
f(-x)



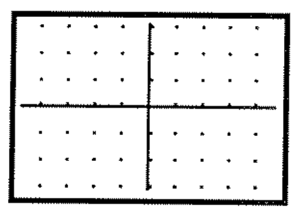
f(x-2)



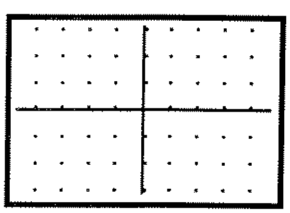
f(x+2)



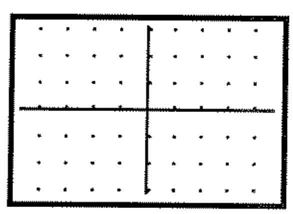
f(x) -3



f(x)+3

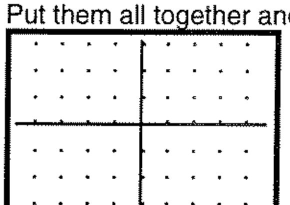


|f(x)|

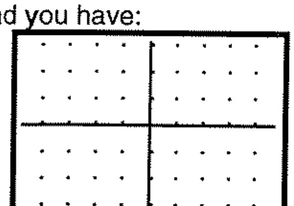


f(|x|)

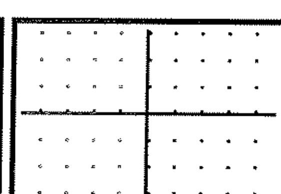
Put them all together and you have:



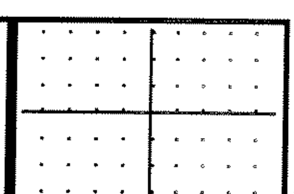
f(x-2) +1



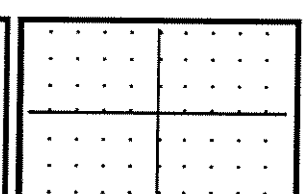
- f(x+1)-2



f(-x) +1



2f(x) -1



|f(x+1)-2|

FAMILIES OF FUNCTIONS 'FOLIO
JUNIOR HONORS PRECALCULUS

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Family of Functions Portfolio – First Semester

On Time	0	5	10
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Completion:			
Graphs	0	5	10
Written Work	0	5	10
Problems	0	5	10
Applications	0	5	10

TOTAL ______ / 70

FAMILY OF FUNCTIONS

$f(x) =$ __________ $f^{-1}(x) =$ __________

Sketch function and inverse here:

Domain

Range

Characteristics

Variations on the function:

$-2f(x) =$ __________ $f(-x)=$ __________ $|f(x)|=$ __________ $f|x|=$ __________

$f(x+1)=$ __________ $f(x) - 2=$ __________ $f(x-2) + 1=$ __________ $f(2x)=$ __________

Describe an application or experiment which illustrates this function. This should be some type of data which might be represented by this type of function.

Create a paragraph or poem about this function. Be clever.

Please complete the other side before handing this in.

Write a word problem using this function and show how to solve it numerically, algebraically and graphically as appropriate.

Problem:

numerically

algebraically

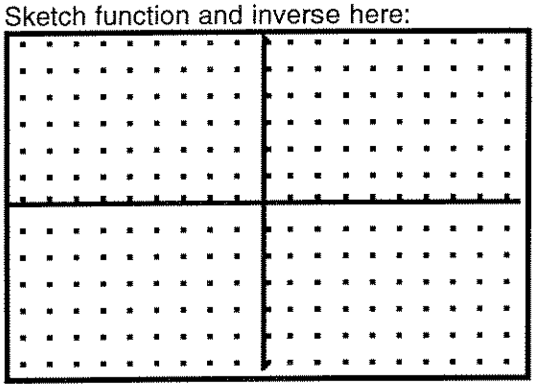
graphically

Write your quick-draw score here and redo those you missed: ______________

______________________ **PERIODIC FAMILY OF FUNCTIONS**

$f(x)$ = ____________ $f^{-1}(x)$ = ____________

Sketch function and inverse here:



Domain

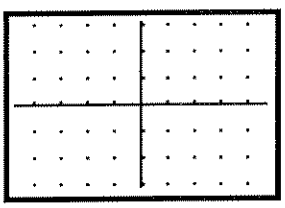
Range

Characteristics

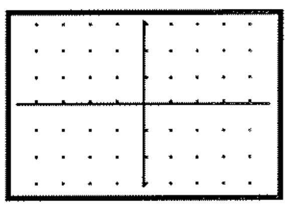
x-scale: $\pi/3$

Variations on the function:

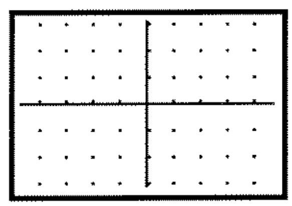
$-2f(x)$ = __________



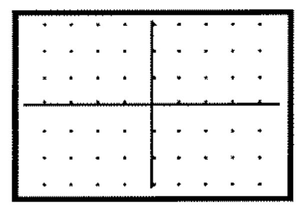
$f(-.5x)$= __________



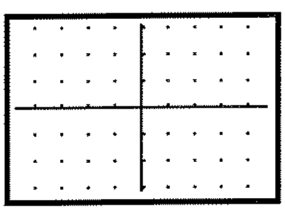
$|f(x)|$= __________



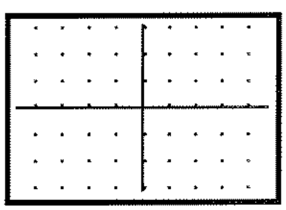
$f|x|$= __________



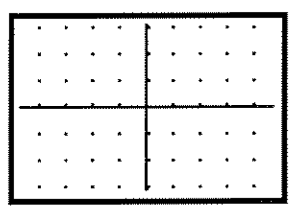
$f(x+\pi/3)$= ________



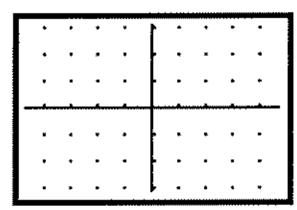
$f(x) - 2$= __________



$f(x-\pi/2) + 1$= _________



$f(2x)$= __________



Describe something which behaves like this function.

Create a paragraph or poem about this function. Be clever.

Please complete the other side before handing this in.

Write a word problem using this function and show how to solve it numerically, algebraically and graphically as appropriate.

Problem:

numerically

algebraically

graphically

Write your quick-draw score here and redo those you missed: ______________

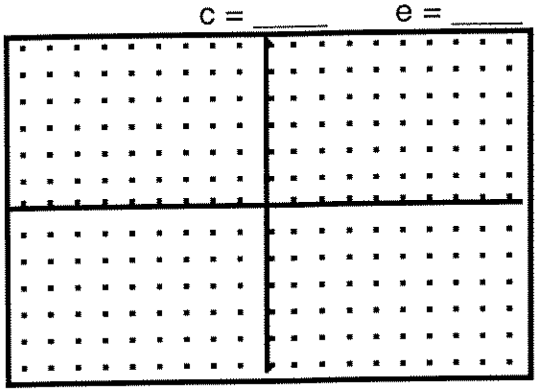
ELLIPTICAL FAMILY OF CONIC RELATIONS

Write each equation and sketch it, including foci and axes.

Vertical______________________________

a=5 , b = 3 (h,k) = (0,0)

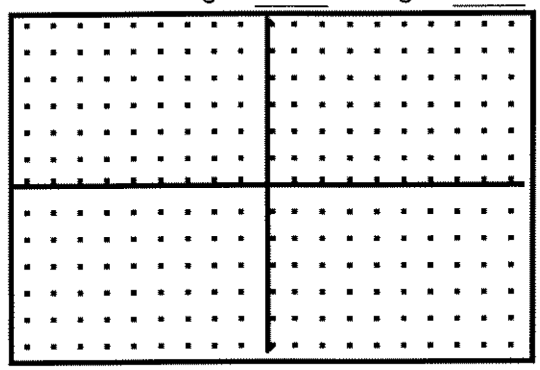
c = _____ e = _____



Horizontal______________________________

a = 5, b = 4 (h,k) = (0,0)

c = _____ e = _____

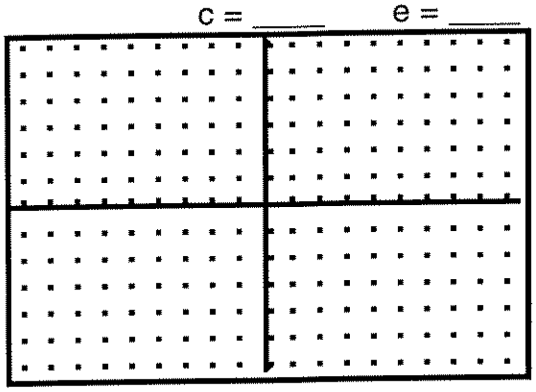


Variations on the relation given {a,b,h,k}

____

Vertical {3,2,-1,2}

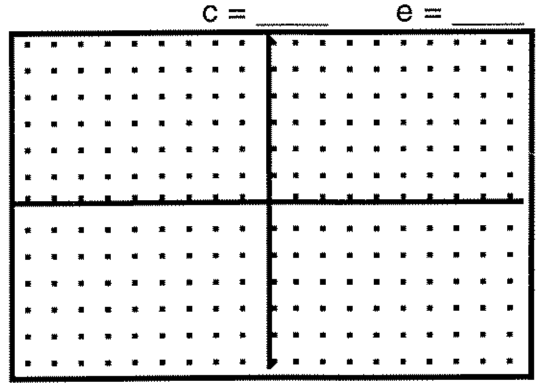
c = _____ e = _____



____

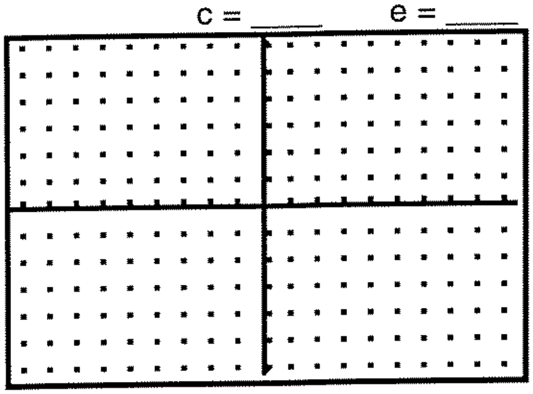
Horizontal {4,1,3,-2}

c = _____ e = _____



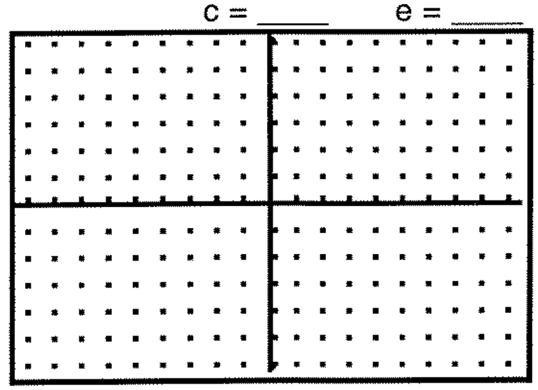
____

c = _____ e = _____



____

c = _____ e = _____



Create a paragraph or poem about this relation. Be clever.

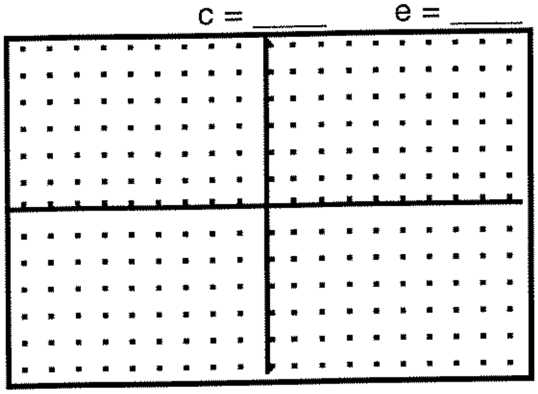
HYPERBOLIC FAMILY OF CONIC RELATIONS

Write each equation and sketch including foci, asymptotes and axes.

Vertical____________________

$a=5$, $b = 3$ $(h,k) = (0,0)$

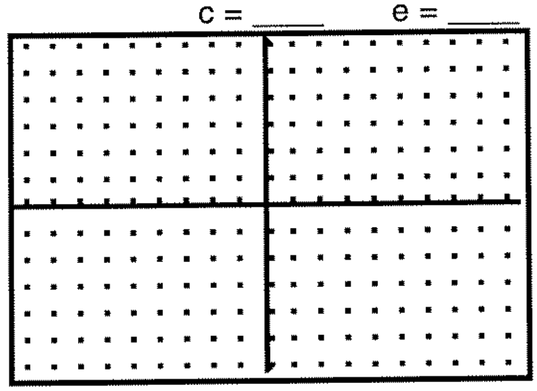
c = ____ e = ____



Horizontal____________________

$a = 3$, $b = 4$ $(h,k) = (0,0)$

c = ____ e = ____

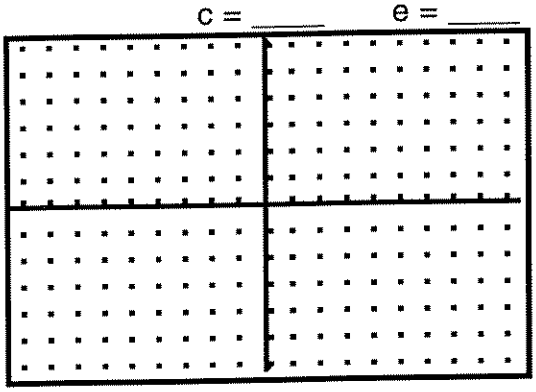


Variations on the relation given {a,b,h,k}

____

VERTICAL {2,3,-1,2}

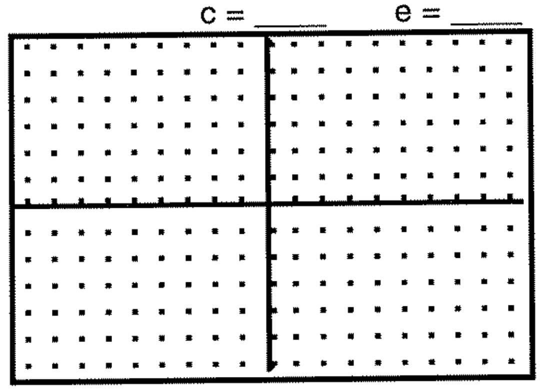
c = ____ e = ____



____

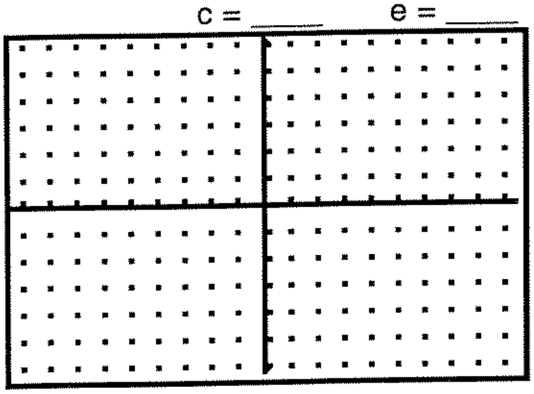
HORIZONTAL {1,4,3,-2}

c = ____ e = ____



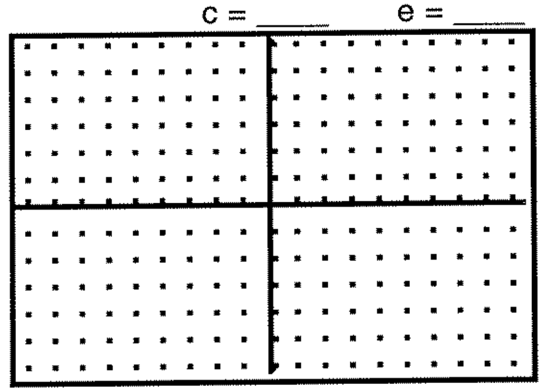
____

c = ____ e = ____



____

c = ____ e = ____



Create a paragraph or poem about this relation. Be clever.

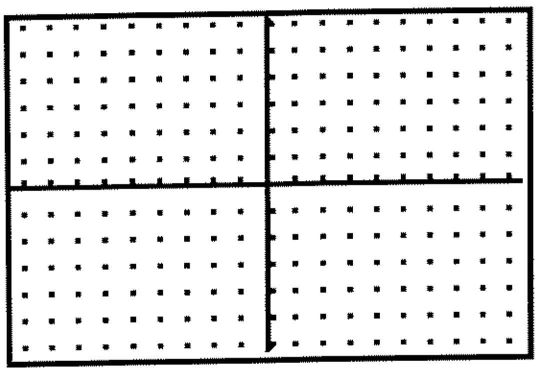
Name____________________
Pd___

PARABOLIC FAMILY OF CONIC RELATIONS

Write the equations. Sketch must include focus, directrix, latus rectum.

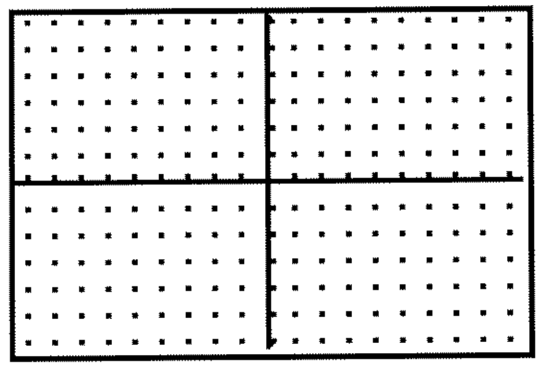
Vertical____________________________

p = -3, (h,k) = (2,3)



Horizontal________________________

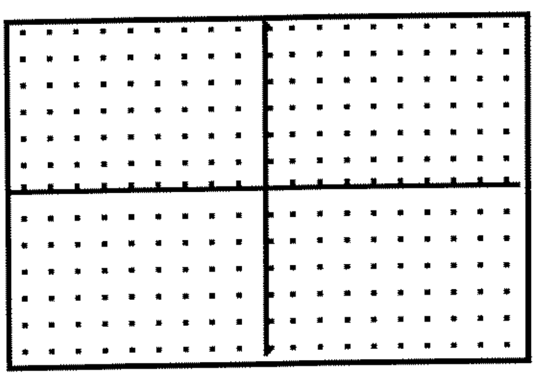
p = .5 (h,k) = (-1,2)



Variations on the relation given {p,h,k}

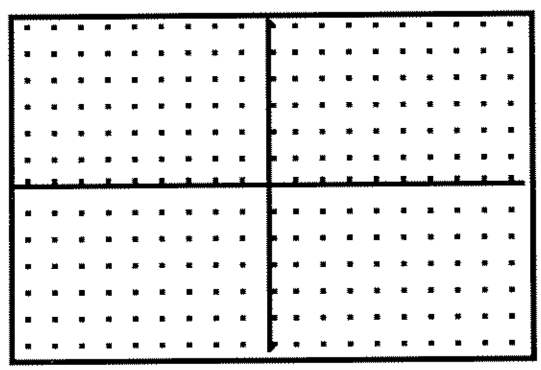
____

{2,-1,2}



____

{-1,3,-2}



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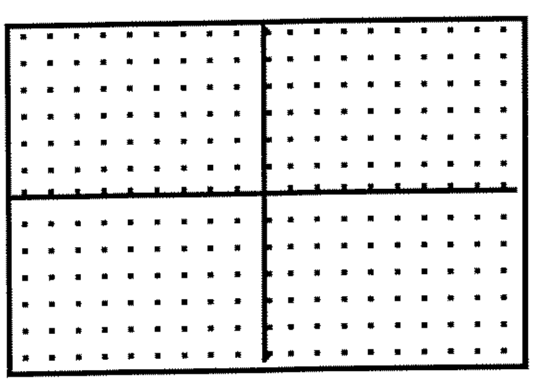

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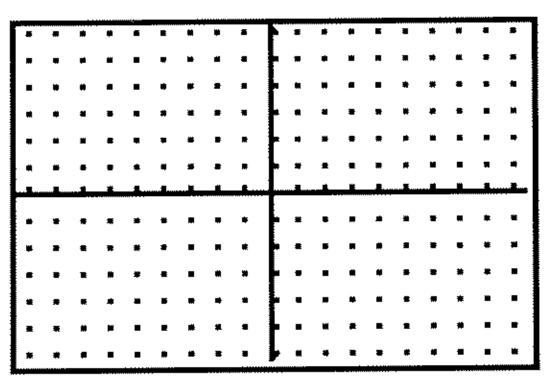


Create a paragraph or poem about this relation. Be clever.

Post chapter one assignment (20 points) A Linear regression

You will collect some interesting data and analyze it using a linear regression.

1. Gather the data.

2. Summarize it in a table, stating the source of your data.

3. Enter it in your calculator and view it as a stat plot. Reproduce the plot accurately on your paper.

4. Perform a linear regression on it and see how well the points fit the line. You might also try a quadratic (p2) regression to see if you can get a better fit. Sketch the results on your graph. Write the equations clearly on the graph.

5. Write a paragraph discussing the slope of your line giving it exact meaning as it relates to your data. You should also discuss the fit of your regression line and your expectations as well as using your line to predict some future value of the data.

Requirements:

You may choose your own data, but it should be two things which are related in some way. One of the variables should be a *function* of the other. You should be able to make a statement such as "the bigger x is, the larger (or smaller) y is." X will be your independent variable, the one on which the other variable depends.

You should have at least eight and no more than fifteen data points.

Make the graph large enough to read and carefully label and title it. It should be neatly drawn and have some personality.

Suggestions of data to use:

Your weight as a function of your height during your younger years.
Your height as a function of your age in months.
The price of a paper back book as a function of the date of publication.
(You should pick books which are similar, but have different publication dates.
The national debt as a function of the year.
The Gross National product as a function of the year.
The value of some home (from tax notices) as a function of the year.
The price of some item as a function of the weight of the item.
(A trip to the grocery store!)
The price of gas as a function of the year (If you can get the data).

*Some great data can be obtained from the Internet, an almanac, your baby book, etc.

Be creative and have fun!

AUTOMOBILE DEPRECIATION

Collect a set of data on a single car model of your choice. It is best to select a model which has been in production for several years. Use the classified ads to find at least ten data points. Each data point will consist of the **age** of the car and the asking price. You may need to use more than one issue of the ads. (Be aware that the two local papers have identical ads on a given day and the best auto ads are on Friday.)

1. Write your data set on paper and store the data points in your calculator.
2. Do a scatterplot on your calculator using age as the independent variable(x).
3. Sketch your extimate of a "best-fit" line and write the equation.
4. Calculate a linear regression and graph the regression line on your scatterplot.

Sketch all of the above items on graph paper. (Be sure to label everything.)

Write a paragraph which includes the following:

- Explain the meaning of the slope of each line.
- Predict the age at which each car will be completely depreciated.
- Project the value of your car two years from now.
- Project how old the car would be if you wanted to spend $10,000 for it.
- Project the price of the car when new. Refer to the paper to find the price of the car when new and compare.

Hand in: Car ads from paper (including one for the new car), table of values, graph including all information and written paragraph. It would be nice if this all fit on two sides of one sheet of paper.

QUADRATIC FUNCTIONS: The falling bodies

The equations for an object thrown straight up in the air and acted upon only by gravity is :

$S(t) = -16t^2+v_o t+h_o$ where v_o = the initial velocity (ft/sec) and
h_o = the initial height (ft)

S represents the height in feet above the ground of the object.
t represents the time in seconds the object has been in the air.

Solve these graphically, draw the graph and read the solutions as accurately as possible without zooming in.

1. A guy lying on the ground on his back shoots an arrow straight up in the air. It leaves the bow with a velocity of 30 feet per sec.

a) What is the maximum height it will reach?
b) When will it reach the maximum height?
c) When will it be 50 feet off the ground?
d) How high will it be after 2 seconds?
e) How long does George have to roll over so he will not get hit?
f) Where will it be after 10 seconds?

2. Jerusha is standing on the edge of a deep canyon. She shoots a rocket from a 200-foot high platform, then removes the platform. The velocity given the rocket is 50 feet per second.

a) What is the maximum height it will reach?
b) When will it reach the maximum height?
c) When will it be 300 feet off the ground?
d) How high will it be after 4.2 seconds?
e) At what second could she put her hand out and catch it as it descends? (She is now on the ground.)
f) Where will it be after 10 seconds?
g) If the canyon is 400 feet deep, when will it reach the bottom?

3. An object is dropped from the top of the Sears Tower in Chicago that is 1320 feet tall. Assume each story is about 18 feet.

a. When will it hit the ground?
b. When will Tillie on the 15th floor see it go by?
c. What floor will it pass at 3 seconds?
d. What is the maximum height it reaches?
e. When does it reach its maximum height?
f. Where is it after 9 seconds?

Steffanettatifaliaestairdapolymia is standing on top of a 20-story (15 ft. per floor) building. She stands at the edge of the building and shoots a rocket straight up in the air with a velocity of 54 ft/sec. It comes down directly besides the building.

1. Write the equation for h(t):

2. Draw a complete graph.
   Tell why it is a **complete** graph.

3. What is the best window for this complete graph?

4. What values of t make sense in the problem?

5. What values of h make sense? Graph the problem situation:

6. When does the rocket pass by her?

7. When does it pass by the 8th floor?

8. When does it hit the ground?

9. When is it 100 ft off the ground?

10. Where is it after 3 sec?

11. What is the maximum height it reaches and at what time does it achieve it?

12. How long is it in the air?

13. Where is it after 10 sec?

14. When will it have traveled as many meters as there are letters in her name?

15. If the first stage of the rocket detaches and falls away at the moment of take-off, what is the difference in time from when the first stage hits the ground and the rocket lands?

On another sheet of paper, make up an interesting, different problem using the height function. Ask 10 good questions and give the answers. (A prize if yours is chosen for the quiz!!)

MAXIMUM BOX PROBLEM

The problem:

A 30 cm by 40 cm piece of paper is used to make a topless box by cutting a square from each corner and folding up the sides.

Make a sketch of the problem here:

Numerically:

Fill in this table.

h =	w =	l =	V = l*w*h
4	22	32	
10			
2			
6			
1			
11			
x			

Algebraically:

Write an equation for the volume of the box : V(x) = ___________________________

Graphically:

Sketch a complete graph of the equation.

Sketch and **label** a complete graph of the problem situation.

Use your graph to answer these questions:

a. What values of x make sense in this problem? ______

b. What is the maximum volume possible for the box? ______

c. What size square should be cut to obtain the maximum volume? ______

d. What size box will you get if you cut squares of 2 cm? ______ of 5 cm? ______

e. What size squares must you cut to get a box with a volume of 200 cm^3? ______

f. What happens if you cut squares of 15 cm? ______

If you feel like a challenge, on the reverse side of this, try to do the problem for a briefcase type box.

Rational function application: Constant volume exercise.

- Students will be in groups of 2 or 3
- Each group will bring between 1 and 2 cups of some substance, which is dry and easily poured, like rice, small beans, etc.
- A variety of cylinders will be available.
- Students will pour their substance into each cylinder measuring the radius of the cylinder and the height to which the cylinder is filled.
- They will plot the height as a function of the radius.
- They will predict an equation, which fits the data.
- They will do a regression on their calculator, coming up with the best fit.
- They will analyze the results coming up with an equation of the form height = $\frac{v}{r^2}$ where v is the volume of their substance.

Radical function: The pendulum

- ⇒ Students in groups of 2-3 given string, weight, tape measure and timer
- ⇒ They will swing the pendulum ten times and time it. Thus by dividing by 10 they will determine the time one swing takes (the period.)
- ⇒ Students will plot length of pendulum (x) and the period (y).
- ⇒ The regression equation should come out to be Period = $2\pi\sqrt{\frac{length}{gravity}}$

Exponential project: Inflation, how it does go up!

- Students will be in groups of 2 or 3.
- The data points will consist of the price of some item over the last few decades. These might be books, magazines, their own home, ski passes, tuition, etc.
- Compare the rise in this price to the consumer price index for that type of item.
- Consumer price data may be found in almanacs or on the Internet.
- They will need at least six data points spanning at least 30 years.
- Plot several items (year, price) including the CPI, performing exponential regressions on them.
- The writing will be a newspaper article to accompany the pictures.

Logarithmic function: Modems **** Thanks to Ray Barton**

- Data points: x = Modem speed, y = Modem price (1200,39), (2400,89), (9600,269), (14400,299) Data from 6-7 years ago. Source: PC magazine curve: $y = 110\ln(x)-754$.
- These data fit a logarithmic curve. It might be interesting for the students to discuss why.
- Students could then research current modem prices and see if they are still logarithmic.
- Other ideas might be computer speed vs. price or memory vs. price.
- Describe what the shape of this graph tells them about technology.

Sinusoidal project: The weather

- Students will be in groups of 3
- Data will be researched on the Internet.
- They may select to do the temperature at several altitudes, the temperature at different latitudes, the length of day at various latitudes or some other thing they think will vary in a sinusoidal way.
- They must graph at least two years worth of data with at least twelve data points per year.
- Each person will graph different data and calculate a linear regression on it.
  - For example, one might do the average monthly temperature in Maine, one the average temperature in Hawaii and one the average temperature in Salt Lake City.
- The three graphs will be combined into one poster (half-poster board in size)
- The poster will have written information about what this shows. They should use words like "amplitude", "period" and shift in the explanation.
- You will be graded on the uniform look of the poster as well as the accuracy of your information. Be sure to remind them to label all axes clearly using words and numbers.

Sequences: The bounce ratio of a ball   CBR may be used

- Three to four students per group: A bouncer, a reader a meter-stick holder.
- Select a sports ball: golf, tennis, basket, Ping-Pong, etc.
- Drop it from a height of 250 cm and measure how high it goes on the first bounce, the second bounce, etc. up to five if you can.
- Get the same measurements dropping it from 200 cm.
- Plot the height as a function of the bounce number each on a separate graph. Find a regression, which fits the data. (It should be $a*r^n$ where a is the initial height and r is the bounce ratio of that ball.)
- In a sports dictionary look up the legal bounce ratio of that ball and see if your data predicts that.
- This should be graphed as a sequence as you do not have the 1.5 bounce of a ball.
- Another interesting graph here is the height of the first bounce as a function of the height of the first drop. This is linear and the bounce ratio is the slope of the line.

Group names______________________

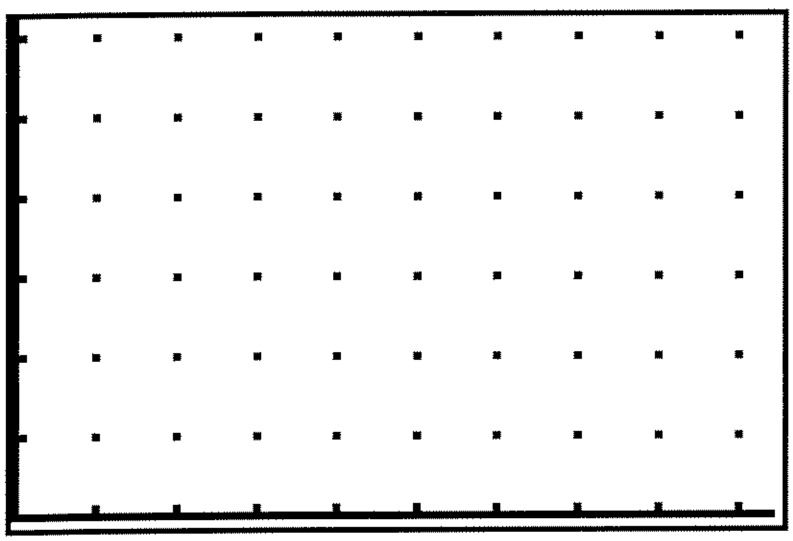
____

Due date _____ pd. _____

Summarize your constant volume data here:

radius											
height											

Equation__________________________



- Compare your equation with that of another group. How are they alike? How are they different? Is there a way to look at the equation and be certain of the volume of substance you used?

- Predictions:

  1. At what radius will the height be exactly 10 cm.? ____ 3 cm.? ____

  2. What will the height be for the following radii?

  5 cm_____ 13 cm_____ 2 cm ______

- Summary: Write a paragraph describing the relationship of theheight to the radius. Describe the mathematics of it and tell how your equation fits in with rational/radical functions we have been studying.

AS THE PENDULUM SWINGS

The period of a pendulum changes with its length. In this activity you measure the period of oscillation for a pendulum as its length is varied. Then, you graph the data to see how the change in length changes the period.

1. Tie a weight to a string.
2. Measure from the center of the weight to the given distance on the string.
3. Swing your weight (pendulum) and record the time required for the pendulum to make 10 full swings (back and forth).
4. Divide the time by 10, that will be the period for that pendulum (the time required for one complete cycle).
5. Put the data in your calculator letting x be the length of the string and y be the period of the pendulum.
6. Look at a scatter plot of your data and try to write an equation which fits the points.
   Hint: this data is not linear nor polynomial!
7. Use your equation to predict.
8. Sketch your equation on the back of this page.
9. Write a paragraph for a textbook explaining how this function relates to this experience.

Summarize your experiment on the back of this page.

Names: ____________________

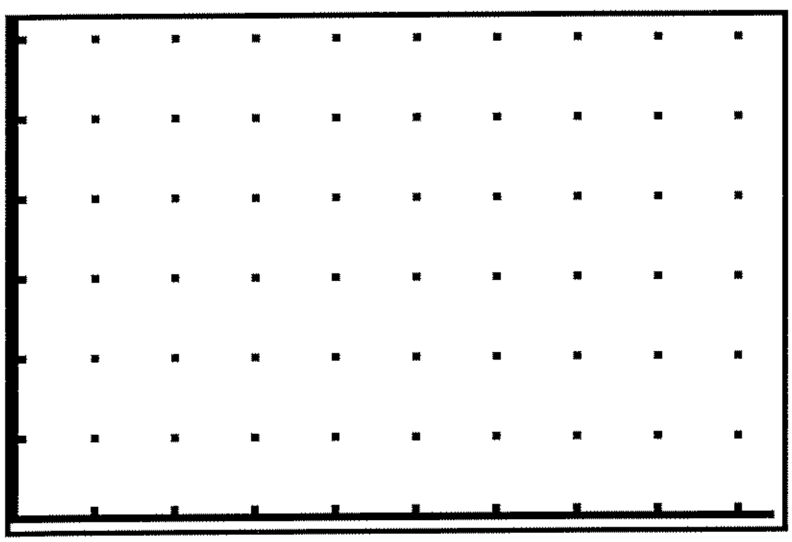
____________________ Pd.______

Due:__________

This table may help organize your data.

length	time for 10 swings	period (t/10)
10 cm		
20 cm		
40 cm		
80 cm		
100 cm		
120 cm		
140 cm		

Equation__________________________



- Predictions:

1. At what length will the period be exactly 1 second? ____ 1.5 sec.? ____

2. What will the period be for the following lengths?

    50 cm_____ 130 cm_____ 200 cm ______

- Summary: Write a paragraph describing the relationship of the period of a pendulum to its length. Describe the mathematics of it and tell how your equation fits in with rational/radical functions we have been studying.

MINI-PROJECT: EXPONENTIAL GROWTH

Objective: You will collect a set of data which has an exponential growth pattern, such as prices which are subject to inflation over time or the growth of some population. You will present your findings in the form of a WELL-WRITTEN newspaper article including appropriate graphics.

Guidelines: This project may be done by a group of up to three people. You must have one set of data for each person in the group, in addition to the data for a general example.

Suggestions: A group of three people might collect data on: (1) the price of newspapers, (2) the price of paperback books, and (3) the price of a magazine. They must compare all three to the Consumer Price Index for entertainment. Other ideas for data include anything for which the price increases over time, like the price of a house, ski pass, or some item of food or clothing. The attached article will give you some more information about inflation. For a population growth model find the growth of the population of some city or state and compare it with the growth of the population of the world.

Presentation: Your project should contain neatly organized data, spectacular graphs which are properly titled and have clearly labeled equations and axes.

Sources: Internet, almanac, parents' records, old paperback books, and magazines found around the house.

Instructions:

1. Find a set of data (at least 6 points) which demonstrates exponential growth. The data should span at least three decades.

2. Put the data in your calculator and fit it with an exponential regression equation.

3. Use your regression to extrapolate (predict some values for future years) and interpolate (find some values not represented by your data but within the range of years you found). Be sure to use y to predict x and x to predict y.

4. Compare your data with inflation using the Consumer Price Index, if it is an item related to inflation, or to the population growth of the world, if it is growth data.

5. Write your newspaper article and prepare your graphs using TI-Graphlink, if you choose.