Math 6770 Homework Exercises

November 1, 2006

1 Homework Set 1

Due Oct. 5, 2006.

1. Consider a birth-death process in which the death rate β is constant ($k_n^- = n\beta$) and the birth rate is population size dependent, say $k_n^+ = \alpha(N-n)$.

   (a) Write the master equation for this process.

   (b) Find the equation for the generating function for this process.

   (c) Show that if $\beta = 0$, the generating function has a stationary solution $g(z) = z^N$. What does this mean?

   (d) Simulate this process using the Gillespie algorithm.

   (e) Simulate the master equation, starting with a population size of one (take $\beta = 1$ for simplicity). Estimate the probability that the population will go extinct for different values of α.

2. Simulate (with the Gillespie algorithm) the three state sodium channel model

$$I \overset{\delta}{\longleftarrow} C \underset{\beta}{\overset{\alpha}{\rightleftarrows}} O \overset{\gamma}{\longrightarrow} I \tag{1}$$

   using parameters $\alpha = 1/\text{ms}$, $\beta = 0.4/\text{ms}$, $\gamma = 1.6/\text{ms}$ and $\delta = 1/\text{ms}$. Collect data from a large number of simulations and use this data to estimate parameters of the model.

2 Homework Set 2

Due Nov. 21, 2006

1. Find the mean first exit time from position $x = 0$ from the piecewise linear potential

$$U(x) = \begin{cases} -\frac{\Delta G x}{L} & -L < x < 0 \\ \frac{\Delta G x}{L} & 0 < x < L \end{cases}, \tag{2}$$

with a reflecting boundary at $x = -L$ and absorbing boundary at $x = L$.

2. Suppose that there are three states underlying the random variable X (with values $x = 0, 1, 2$) with transitions

$$S_0 \underset{\beta}{\overset{2\alpha}{\rightleftarrows}} S_1 \underset{2\beta}{\overset{\alpha}{\rightleftarrows}} S_2. \tag{3}$$

(a) Use the Gillespie algorithm to find sample paths for the solution of

$$\frac{dy}{dt} = x \tag{4}$$

Use this to estimate the mean and the variance as a function of time for this process.

(b) What is the Master equation for this stochastic differential equation?

(c) Suppose that α and β are much greater than 1. Use the rapid equilibrium approximation to find the Fokker Planck equation and the equivalent Langevin equation for this process. Compare the result of this calculation with the numerical estimates found in part a.