Math 6730 Course Outline

1. Asymptotic Expansions - Definitions

   (a) scaling, order relations, roots of equations

2. Regular Perturbation Theory

   (a) Implicit function Theorem

   (b) Power series expansions -

      - Gravitational trajectory (Holmes)
      - perihelion of Mercury (JK)
      - Melnikov function?? (JK)

3. Singular Perturbation Theory

   (a) Example: What is a singular perturbation problem? Damped harmonic oscillations

   (b) Matched Asymptotic Expansions

      i. Boundary value problems
         - uniform expansions, matching principle
         - Boundary layers, interior layers, corner layers
      ii. Initial Value problems
         - QSS, initial layers
         - Michaelis-Menten- rapid equilibrium,
         - fast-slow systems
         - adiabatic reduction for stochastic processes Master equations
      iii. Interior layers, rapid trnsitions
         - FHN/van der Pol
         - traveling waves, fronts and backs

   (c) Multi-scale analysis

      i. Two-time scales, averaging
         - forced oscillations, nonlinear resonance, Duffing, vdPol, Hopf bifurcation, Ginzberg-Landau eqn, Mathieu's eqn
         - coupled phase oscillators, phase locking
         - adiabatic invariance
      ii. Homogenization
         - Effective diffusion/conductance
         - Bidomain equations
         - porous medium equation