Homework Exercises for Mathematics 6710 - Fall 2010

Chapter 1 (due Sept. 27, 2010):

Remark: The use of Maple (or Mathematica) is permitted for any of these exercises as a tool (for example to find eigenvalues and eigenvectors), as long as you demonstrate an understanding of what is being done.

1. Section 1; 3, 4, 9, 10
2. Section 2; 1, 5, 6, 10
3. Section 4; 1, 2, 4
4. Section 5; 1a-d, 3, 7, 13

Chapter 2 (due Oct. 25, 2011):

1. Section 1; 11.
2. Section 2; 1, 2, 4, 6, 15, 26
3. Use the Galerkin method to approximately solve the boundary value problem

$$u'' - u = e^{-x}, \qquad u(0) = 0, \qquad u(1) = 0 \tag{1}$$

   using the two different sets of basis functions $\{\phi_n(x)\}$:

   (a) $\phi_n(x) = \sin(n\pi x)$,

   (b) The cubic splines $S^h(3, 1)$.

   Compare the two solutions you find, using solutions with the same number of basis functions, say 6. Which of the two approximations is better? (Which gives a smaller L_2 error?)

Chapter 3 (due Nov. 24, 2011)

1. Section 2; 3
2. Section 3; 2
3. Section 4; 2, 3
4. Section 5; 2

5. One way to approximate the integral equation

$$u(x) + \int_{\infty}^{\infty} k(x,\xi)u(\xi)d\xi = f(x) \tag{2}$$

is with the pointwise approximation

$$u(x_j) + \sum_k k(x_j, x_k)u(x_k)d\xi = f(x_j) \tag{3}$$

where $x_j = (j-N)h, j = 0, 1, \cdots, 2N$.

(a) Show that this approximation can be derived using a projection built with sinc functions.

(b) Under what conditions on $k(x,\xi)$ does this approximation converge to the solution of the integral equation, in the limit $N \to \infty$, with h fixed?

6. (a) What is the appropriate projection operator for the Galerkin method using the cubic splines? That is, define P such that $P : H \to S^h(3,1)$, and $P^2 = P$.

(b) Find the cubic spline approximation for the solutions of problems Section 3.2; 3a,b. (What are the approximating equations? Solve these equations in the simple case that $h = \frac{1}{2}$. How does the approximation solution compare with the exact solution?

Chapter 4 (Due Dec. 16, 2011)

1. Section 1; 5, 7
2. Section 2; 2, 4, 10
3. Section 3; 3, 9, 11
4. Section 4; 2, 5, 8
5. Section 5; 5, 12