Homework 2: Due Feb. 26, 2015

1. A particle diffuses in a region $-L < x < L$ starting from $x = 0$. The particle can degrade with degradation rate $\gamma$. The goal of this exercise is to determine the probability that the particle reaches one of the boundaries before it decays.

(a) The answer should be a function of what single non-dimensional parameter?
(b) Numerically simulate this problem to get a numerical estimate of the answer for several parameter values.
(c) The probability that the particle decays before reaching the boundary is $\pi(0)$ where $\pi(x)$ satisfies the differential equation

$$D \pi_{xx} - \gamma \pi = -\gamma, \quad \pi(-L) = 0, \quad \pi(L) = 0 \quad (1)$$

Calculate the probability that it reaches the boundary before decaying and compare with what you found from your numerical simulation.

2. A dingo population which lives in the eastern parts of Australia is prevented from invasion to the west by a fence that runs north-south. Imagine that the fence breaks somewhere at time $t = 0$. A farm is located on the west side of the fence, exactly 100 miles west of the hole in the fence. The farmers would like to know how long it will take the dingoes to reach their farm. Model the spread of dingoes as:

$$u_t = Du_{xx} + ku(1 - \frac{u}{K}) \quad (2)$$

with $k = 1$ (1/month), and $K = 1$ (in units of $u$).

(a) The region between the fence and farm is flat and the diffusion constant $D_1 = 100$ (miles$^2$/month). When does the dingo population reach the farm? (consider a traveling wave and calculate the wave speed)
(b) How will the time change if the diffusion constant is $D_2 = 50$ miles$^2$/month due to some rock and slope in the region?