Math 5110 - Fall 2007
Homework Problem Set 8
Due Nov. 29, 2007

1. Consider the system of linear differential equations

$$\frac{dx}{dt} = y - 3x, \qquad \frac{dy}{dt} = 100(2x - y) \tag{1}$$

subject to initial conditions $y(0) = 0$, $x(0) = 1$

   (a) Simulate the solution of this system numerically.

   (b) For this system, which is the fast variable and which is the slow variable?

   (c) What is the quasi-steady state approximation for this system?

   (d) What are the appropriate initial conditions for the quasi-steady state approximation? (Before the quasi-steady state approximation becomes valid, there is rapid transient during which one variable changes quickly while another is essentially constant. What is the end result of this rapid process?)

   (e) Compare the numerical solution of the full system of equations with the quasi-steady state approximation. Where do they agree and where do they disagree?

2. Suppose that an enzyme E has a reactive site that can bind the substrate S, but there is also another chemical I (an inhibitor) that can bind with the enzyme to prevent with reaction with S. The chemical kinetics are

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C_1 \xrightarrow{\kappa_1} P + E,$$
$$E + I \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} C_2$$

   (a) Use the law of mass action to write balance equations for all the reactants.

   (b) What combination of variables involving enzyme is conserved? Use this conservation law to eliminate E from the equations.

   (c) Suppose that the binding and unbinding of E with S or I is fast. Make a quasi-steady state approximation to find equations for C_1 and C_2 in terms of S.

   (d) Find a simplified differential equation for the rate of production of P by using the quasi-steady state approximation from the last question.

3. In Figure 1 is shown the state diagram for an L-type calcium channel.

   (a) Use the law of mass action to write differential equations for the probability of being in each of the states. Is there a conserved quantity?

   (b) Suppose the rate constants α and β are much larger than all other rate constants. What reactions should be in quasi-equilibrium?

   (c) Use the quasi-equilibrium approximation to find equations for the combined states $N = N_1 + N_2$ and $C = C_1 + C_2$. The reduced reaction diagram is shown in Figure 2. What are the reaction rates a, b, and F?

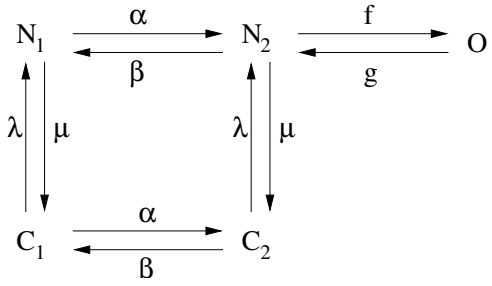

Figure 1:

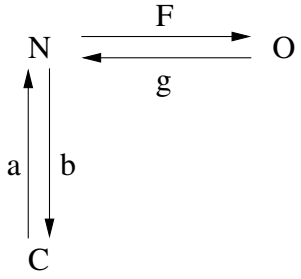

Figure 2: