Math 5110 - Fall 2007
Homework Problem Set 4
Due Oct. 2, 2004

1. The differential equation

$$\frac{dN}{dt} = rN\ln(\frac{K}{N}) \tag{1}$$

is called the Gompertz differential equation. Use the transformation $u = \ln N$ to transform this equation into a linear differential equation and then use this to solve the equation for N.

2. Suppose the population size of some species of organism follows the model

$$\frac{dN}{dt} = \frac{3N^2}{2+N^2} - N \tag{2}$$

   (a) Find the steady states.

   (b) Draw the "phase-line" diagram.

   (c) Which of the equilibria are stable and which are unstable?

   (d) Interpret these results in biological terms. Why might this population behave as it does for small values?

3. Analyze the behavior of the population model

$$\frac{du}{dt} = ru(1-\frac{u}{K}) - \frac{u^2}{1+u^2} \tag{3}$$

Find the different behavior when there is population size dependent growth (K finite) and when there is no such size dependence ($K \to \infty$). Explain why the behaviors are different.