11.4 # 17

Find eqn of the plane through $(-1,-2,3)$ and perpendicular to both the planes $x-3y-2z=7$, $2x-2y-z=-3$

perpendicular to each plane is their normal

$$\vec{n}_A = \vec{i} - 3\vec{j} + 2\vec{k} \qquad \vec{n}_B = 2\vec{i} - 2\vec{j} - \vec{k}$$

the new plane's normal will be perpendicular to the normals of the other planes so that

$$\vec{n}_{New} = \vec{n}_A \times \vec{n}_B$$

$$= \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 2 & -2 & -1 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ -2 & -1 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & -3 \\ 2 & -2 \end{vmatrix}\vec{j}$$

$$= 7\vec{i} + 5\vec{j} + 4\vec{k}$$

using the point $(-1,-2,3)$ we have the equation:

$$7(x+1) + 5(y+2) + 4(z-4) = 0$$

$$\boxed{7x + 5y + 4z = -5}$$

11.4 #32 prove $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$

$\vec{u} = \langle u_1, u_2, u_3 \rangle \quad \vec{v} = \langle v_1, v_2, v_3 \rangle \quad \vec{w} = \langle w_1, w_2, w_3 \rangle$

$$\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \langle v_1 + w_1, v_2 + w_2, v_3 + w_3 \rangle$$

$$= \langle u_2(v_3+w_3) - u_3(v_2+w_2), u_3(v_1+w_1) - u_1(v_3+w_3), u_1(v_2+w_2) - u_2(v_1+w_1) \rangle$$

$$= \langle u_2v_3 + u_2w_3 - u_3v_2 - u_3w_2, u_3v_1 + u_3w_1 - u_1v_3 - u_1w_3, u_1v_2 + u_1w_2 - u_2v_1 - u_2$$

$$= \langle (u_2v_3 - u_3v_2) + (u_2w_3 - u_3w_2), (u_3v_1 - u_1v_3) + (u_3w_1 - u_1w_3),$$

$$(u_1v_2 - u_2v_1) + (u_1w_2 - u_2w_1) \rangle = \boxed{(\vec{v} \times \vec{u}) + (\vec{w} \times \vec{u})}$$

11.5 #23

find $\vec{v}$, $\vec{a}$ and speed at $t_1 = 2$ for $\vec{r}(t) = \vec{i} + \left(\int_0^t x^2 dx\right)\vec{j} + t^{2/3}\vec{k}$

$\vec{v}(t) = \vec{r}\,'(t) = t^2\vec{j} + \frac{2}{3}t^{-1/3}$

$\vec{a}(t) = \vec{v}\,'(t) = 2t\vec{i} - \frac{2}{9}t^{-4/3}\vec{k}$

$$\boxed{\begin{aligned}\vec{v}(2) &= 4\vec{j} + \frac{2}{3\sqrt[3]{2}}\vec{k} \\ \vec{a}(t) &= 4\vec{i} - \frac{-2}{9(\sqrt[3]{2})^4}\vec{k}\end{aligned}}$$

$S = \|\vec{v}(t)\| = \sqrt{t^4 + \left(\frac{2}{3}t^{-1/3}\right)^2} = \sqrt{t^4 + \frac{4}{9}t^{-2/3}}$

$$\boxed{S(2) = \sqrt{16 + \frac{4}{9(\sqrt[3]{2})^2}} \cong 4.035}$$

11.5 # 32

$(\rightarrow)$ if $\|\vec{r}(t)\| = \text{constant}$ then $\|\vec{r}(t)\|^2 = \vec{r}\cdot\vec{r} = \text{constant}$

$\frac{d}{dt}(\vec{r}\cdot\vec{r}) = \frac{d}{dt}(\text{constant})$

$\vec{r}\cdot\frac{d\vec{r}}{dt} + \vec{r}\cdot\frac{d\vec{r}}{dt} = 0$

$2\vec{r}\cdot\frac{d\vec{r}}{dt} = 0$

$\vec{r}\cdot\frac{d\vec{r}}{dt} = 0$ ✓

$(\leftarrow)$ if $\vec{r}\cdot\frac{dr}{dt} = 0 \Rightarrow 2\vec{r}\cdot\frac{d\vec{r}}{dt} = 0$

but $2\vec{r}\cdot\frac{d\vec{r}}{dt} = \frac{d}{dt}(\vec{r}\cdot\vec{r})$

so $\frac{d}{dt}(\vec{r}\cdot\vec{r}) = 0$

which means $\vec{r}\cdot\vec{r} = \text{constant}$

$\Rightarrow \|\vec{r}\|^2 = \text{constant}$

$\Rightarrow \|\vec{r}\| = \text{constant}$ ✓

11.6 # 44

consider $\vec{r}(t) = \sin t\,\vec{i} + \cos t\,\vec{j} + (t^2 - 3t + 2)\vec{k}$

- $\vec{k}$ component measures height from ground
- $t \geq 0$

a) does particle move downward?

if the derivative of the $\vec{k}$ component is negative, particle moving downward

$2t - 3 < 0$ for $\boxed{0 \leq t \leq \frac{3}{2}}$

b) does particle ever stop moving?

if it stops, speed $= 0$. $\vec{v}(t) = \cos t\,\vec{i} - \sin t\,\vec{j} + (2t-3)\vec{k}$

speed $= \|\vec{v}(t)\| = \sqrt{\cos^2 t + (-\sin t)^2 + (2t-3)^2}$

$0 = \sqrt{1 + 4t^2 - 12t + 9}$

$0 = \sqrt{4t^2 - 12t + 10} \Rightarrow 4t^2 - 12t + 10 = 0$

any solutions?

$t = \dfrac{12 \pm \sqrt{(-12)^2 - 4(4)(10)}}{2(4)}$

$= \dfrac{12}{8} \pm \dfrac{\sqrt{144 - 160}}{8}$ NO real solutions

speed never zero, particle doesn't stop moving

c) at what time does it reach 12 m above ground

$\left.\begin{array}{l} t^2 - 3t + 2 = 12 \\ t^2 - 3t - 10 = 0 \end{array}\right\}$ $(t+2)(t-5) = 0 \Rightarrow t \geq 0$

so $\boxed{t = 5}$

d) what is velocity at 12 m above ground?

$\boxed{\vec{v}(5) = \cos 5\,\vec{i} - \sin 5\,\vec{j} + 7\vec{k}}$

11.6 #16

find symmetric equations of line through $(2,-4,5)$ parallel to the plane $3x+y-2z=5$ and perpendicular to the line

$$\frac{x+8}{2} = \frac{y-5}{3} = \frac{z-1}{-1}$$

$\langle 2,3,-1\rangle$ is the direction vector perpendicular to the line we seek

we want the vector perpendicular to $\langle 2,3,-1\rangle$ and perpendicular to the normal of the plane $3x+y-2z=5$, which is $\langle 3,1,-2\rangle$

$$\vec{v} = \langle 3,1,-2\rangle \times \langle 2,3,-1\rangle$$

$$= \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 2 & 3 & -1 \end{vmatrix} = \langle 5,-1,7\rangle$$

CAN ALSO SWAP THESE, $\langle 2,3,-1\rangle \times \langle 3,1,-2\rangle$

now use this as the direction vector, and $(2,-4,5)$ as the point :

$$x = 2+5t, \quad y = -4-t, \quad z = 5+7t$$

$$\Rightarrow \quad \boxed{\frac{x-2}{5} = \frac{y+4}{-1} = \frac{z-5}{7}}$$

11.6 # 23

Find the symmetric equations of the tangent line to the curve w/ equation

$$\vec{r}(t) = 2\cos t\,\vec{i} + 6\sin t\,\vec{j} + t\,\vec{k} \quad \text{at } t = \frac{\pi}{3}$$

need a point on the tangent line and a vector parallel to the tangent line.

a point: $\vec{r}\left(\frac{\pi}{3}\right) = 2\cos\left(\frac{\pi}{3}\right)\vec{i} + 6\sin\left(\frac{\pi}{3}\right)\vec{j} + \frac{\pi}{3}k$

the point is $\left(1, 3\sqrt{3}, \frac{\pi}{3}\right)$

the vector parallel:

we know the velocity vector is tangent

$\vec{v}(t) = \vec{r}\,'(t) = -2\sin t\,\vec{i} + 6\cos t\,\vec{j} + \vec{k}$

so that at $t = \frac{\pi}{3}$, this vector is

$\vec{v}\left(\frac{\pi}{3}\right) = -\sqrt{3}\,\vec{i} + 3\vec{j} + \vec{k}$

use $\left(1, 3\sqrt{3}, \frac{\pi}{3}\right)$ with $\langle a, b, c\rangle = \langle -\sqrt{3}, 3, 1\rangle$

$$\Rightarrow \boxed{\frac{x-1}{-\sqrt{3}} = \frac{y-3\sqrt{3}}{3} = z - \frac{\pi}{3}}$$

11.7 #33

Find $\kappa, \vec{T}, \vec{N}, \vec{B}$ at $t=0$ if $\vec{r}(t)=e^{-2t}\vec{i}+e^{2t}\vec{j}+2\sqrt{2}t\vec{k}$

$\vec{T}=\dfrac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$ $\quad \vec{N}=\dfrac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$ $\quad \vec{B}=\vec{T}\times\vec{N}$ $\quad \kappa=\dfrac{\|T'(t)\|}{\|r'(t)\|}$

$\vec{r}'(t)=-2e^{-2t}\vec{i}+2e^{2t}\vec{j}+2\sqrt{2}\vec{k}$ $\quad \|r'(t)\|=\sqrt{4e^{-4t}+4e^{4t}+8}$

$\vec{r}'(0)=-2\vec{i}+2\vec{j}+2\sqrt{2}k$

$\|\vec{r}'(0)\|=\sqrt{(-2)^2+2^2+(2\sqrt{2})^2}=\sqrt{4+4+8}=4$

$\vec{T}(t)=\dfrac{-2e^{-2t}\vec{i}+2e^{2t}\vec{j}+2\sqrt{2}\vec{k}}{\sqrt{4e^{-4t}+4e^{4t}+8}}$ $\quad \boxed{\vec{T}(0)=-\frac{1}{2}\vec{i}+\frac{1}{2}\vec{j}+\frac{\sqrt{2}}{2}\vec{k}}$

we can also find $\vec{N}$ using $\vec{N}=\dfrac{\vec{a}-a_T\vec{T}}{a_N}$

$a_T=\dfrac{\vec{r}'(0)\cdot\vec{r}''(0)}{\|\vec{r}'(0)\|} \Rightarrow \vec{r}''(t)=4e^{-2t}\vec{i}+4e^{-2t}\vec{j}$

$=\dfrac{(-2\cdot4+2\cdot4)}{4}=0$

$a_N=\dfrac{\|\vec{r}'(0)\times\vec{r}''(0)\|}{\|\vec{r}'(0)\|}=\dfrac{1}{\|\vec{r}'(0)\|}\left\|\begin{vmatrix} i & j & k \\ -2 & 2 & 2\sqrt{2} \\ 4 & 4 & 0\end{vmatrix}\right\|=\frac{1}{4}\cdot\|\langle-8\sqrt{2},-8\sqrt{2},-16\rangle$

$=4\sqrt{2}$

$\vec{N}(0)=\frac{1}{4\sqrt{2}}\vec{a}(0) \Rightarrow \boxed{\vec{N}(0)=\frac{1}{\sqrt{2}}\vec{i}+\frac{1}{\sqrt{2}}\vec{j}}$

$\boxed{\kappa(0)=\dfrac{\|\vec{r}'(0)\times r''(0)\|}{\|r'(0)\|^3}=\dfrac{\sqrt{2}}{4}}$ (from a_N above)

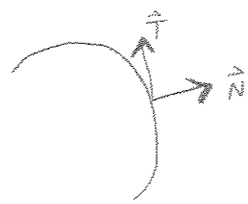
$\vec{B}(0)=\vec{T}(0)\times\vec{N}(0)=\begin{vmatrix} i & j & k \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{2}/2 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\end{vmatrix} \Rightarrow \boxed{\vec{B}(0)=-\frac{1}{2}\vec{i}+\frac{1}{2}\vec{j}-\frac{1}{\sqrt{2}}\vec{k}}$

11.7 # 61

centrifugal force pulling outward must balance the horizontal friction force:

recall, going around a curve at <u>constant speed</u> gives:

$$a_T = \frac{d^2s}{dt^2} = 0, \quad a_N = \left(\frac{ds}{dt}\right)^2 \kappa > 0$$



the normal component must match the friction force:

$$\underbrace{\left(\frac{ds}{dt}\right)^2}_{\text{speed}^2} \underbrace{\kappa}_{\frac{1}{R}} = \mu m g$$

a force by Newtons law is ma which has units of mass $\cdot \frac{\text{length}}{\text{time}^2}$

so we need to include the mass of the car:

$$m\left(\frac{ds}{dt}\right)^2 \kappa = \mu m g$$

$$\cancel{m} v^2 \cdot \frac{1}{R} = \mu \cancel{m} g$$

$$v^2 = R\mu g$$

$$v = \sqrt{R\mu g}$$

$$v = \sqrt{(400\,\text{ft})(.4)(32\,\text{ft}/\text{sec}^2)}$$

$$= \sqrt{5120\,\text{ft}^2/\text{s}^2}$$

$$\boxed{v \cong 71.55\,\text{ft}/\text{sec}}$$