9.6 #20

$$\sum_{n=0}^{\infty} 2^n x^n \qquad \text{use ratio test:} \quad \rho = \lim_{n\to\infty} \left| \frac{2^{n+1} x^{n+1}}{2^n x^n} \right| = \lim_{n\to\infty} |2x| = |2x|$$

$|2x| < 1$ when $-\frac{1}{2} < x < \frac{1}{2}$

check endpoints:

$x = \frac{1}{2}$, $\sum_{n=0}^{\infty} 2^n (\frac{1}{2})^n = \sum_{n=0}^{\infty} 1$ diverges

$x = -\frac{1}{2}$, $\sum_{n=0}^{\infty} 2^n (-\frac{1}{2})^n = \sum_{n=0}^{\infty} (-1)^n$ diverges

convergence set : $(-\frac{1}{2}, \frac{1}{2})$.

9.7 #7

$\frac{x^2}{1-x^4}$ If we let $x = x^4$ in the geometric series:

$$\frac{1}{1-x^4} = 1 + x^4 + x^8 + x^{12} + \ldots \qquad -1 < x < 1$$

multiply by x^2...

$$\frac{x^2}{1-x^4} = x^2 + x^6 + x^{10} + x^{14} + \ldots \qquad -1 < x < 1$$

9.7 # 24 find power series for $\int_0^x \frac{\tan^{-1} t}{t}\, dt$

we know $\frac{\tan^{-1} x}{x} = 1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \ldots$

$$\int_0^x 1 - \frac{t^2}{3} + \frac{t^4}{5} - \frac{t^6}{7} + \ldots\, dt = x - \frac{x^3}{9} + \frac{x^5}{25} - \frac{x^7}{49} + \ldots$$

9.8 #19

find Taylor series in $x-a$ through $(x-a)^3$

$f^{(n)}(x) = e^x$ for all n...

$f(1) = f'(1) = f''(1) = \ldots e$

$$e^x \approx e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 + \ldots$$

9.9 #13

find the Taylor polynomial of order 3 based at a

$\cot^{-1} x$, $a=1$

first find the 1st 3 derivatives... then evaluate at $a=1$

$f(x) = \cot^{-1}(x)$ $\qquad f(1) = \frac{\pi}{4}$

$f'(x) = -\frac{1}{1+x^2}$ $\qquad f'(1) = \frac{-1}{2}$

$f''(x) = \frac{2x}{(1+x^2)^2}$ $\qquad f''(1) = \frac{1}{2}$

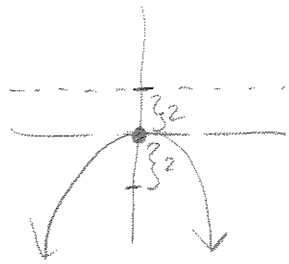
$f'''(x) = \frac{-6x^2+2}{(1+x^2)^3}$ $\qquad f'''(1) = -\frac{1}{2}$

then plug into the formula for $P_3(x)$...

$$P_3(x) = \frac{\pi}{4} - \frac{1}{2}(x-1) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(x-1)^2 - \frac{1}{2}\left(\frac{1}{6}\right)(x-1)^3 + \ldots$$

10.1 #11 standard equation: directrix is $y-2=0$

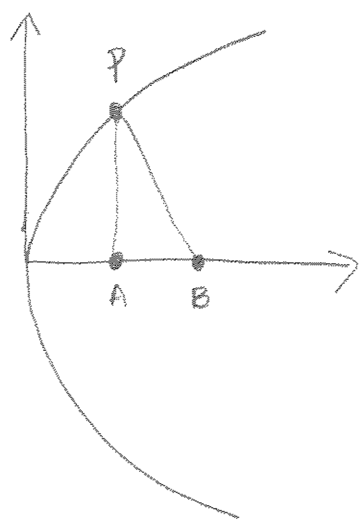
if vertex is at origin, $y=2$ is directrix:



then $p=2 \Rightarrow x^2 = -(4)(2)y$

$\boxed{x^2 = -8y}$

10.1 #38



find $|AB|$

if p is a point $P(x_1, y_1)$

find the equation of the normal line PB

using implicit differentiation

$$2yy' = 4p \Rightarrow y' = \frac{2p}{y}$$

$\frac{2p}{y_1}$ is the slope of the tangent line at P.

$-\frac{y_1}{2p}$ is the slope of the normal line

so the equation is $y - y_1 = \frac{-y_1}{2p}(x - x_1)$ want to find coords of B!

when does the normal line pass through the x axis? when $y=0$

$y=0 \Rightarrow 2p + x_1 = x$ so B has x value $2p + x_1$

and A has the x value x_1

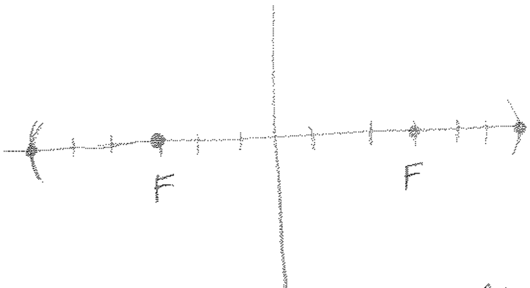
$A = (x_1, 0)$ $B = (2p + x_1, 0)$

$$\boxed{|AB| = 2p}$$

10.2 # 17

Ellipse with focus at $(-3,0)$ vertex at $(6,0)$ centered at $(0,0)$

$\downarrow$ $c=3$ $\downarrow$ $a=6$



$\left.\begin{array}{l} a=6 \\ c=3 \end{array}\right\} b=\sqrt{36-9}=\sqrt{27}$

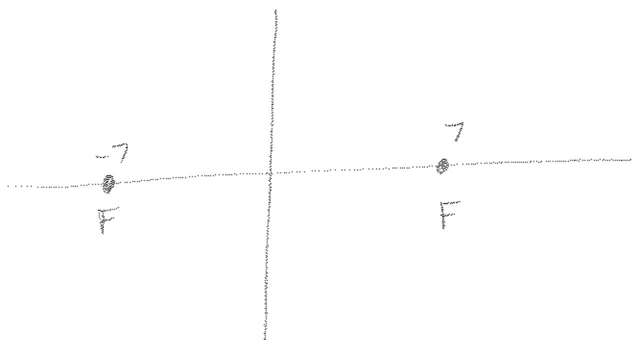
equation is $\boxed{\dfrac{x^2}{36}+\dfrac{y^2}{27}=1}$

10.2 # 33

the difference of the distances of P from $(\pm 7, 0)$ is 12

the foci are at $(\pm 7, 0)$

tells us this is a hyperbola



we know $2a$ = this difference

$2a = 12$

$a = 6$

and for a hyperbola $b=\sqrt{c^2-a^2}=\sqrt{13}$

$\boxed{\dfrac{x^2}{36}-\dfrac{y^2}{13}=1}$

10.3 #31

Find the foci of the ellipse

$$16(x-1)^2 + 25(y+2)^2 = 400$$

$$\Rightarrow \frac{(x-1)^2}{25} + \frac{(y+2)^2}{16} = 1$$

$a = 5 \quad b = 4 \quad \Rightarrow$ horizontal ellipse centered at $(1,-2)$

$$c = \sqrt{25-16} = 3$$

so the foci are at $(1 \pm 3, -2)$

$$\boxed{\text{Foci: } (-2,-2), (4,-2)}$$