USE the chain rule to find

(R1) find $\frac{df}{dt}$ if $f(x,y) = x\ln(y)$ where $x = t^2$ and $y = e^t$

Check

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= \ln(y)(2t) + \frac{x}{y}(e^t)$$

$$= \ln(e^t)2t + \frac{t^2}{e^t}e^t$$

$$= 2t^2 + t^2$$

$$= 3t^2$$

(R2) #18 pg 651

$T = e^{-x-3y}$ we wand $\frac{dT}{dt}$ at the origin if $\frac{dx}{dt} = 2$, $\frac{dy}{dt} = 2$

$$\frac{dT}{dt} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt}$$

$$= -e^{-x-3y}(2) - 3e^{-x-3y}(2)$$

$$= -8e^{-x-3y}$$

at $(0,0)$ $\frac{dT}{dt} = 0$.

R3 Find the shortest distance from the origin to the plane $x + 2y + 3z = 12$

want to minimize d or d^2

$d^2 = x^2 + y^2 + z^2$ if $x = 12 - 2y - 3z$

then $f(y,z) = (12 - 2y - 3z)^2 + y^2 + z^2$

$\nabla f = \langle -48 + 12z + 10y, -72 + 12y + 20z \rangle$

at $\langle 0, 0 \rangle = \nabla f$

$12z + 10y = 48$
$20z + 12y = 72$ $\Rightarrow$ $\left(\frac{12}{7}, \frac{18}{7}\right)$

$D = f_{yy} f_{zz} - f_{yz}^2 = (10)(20) - (12)^2 = 200 - 144 = 56 > 0$

$f_{yy} = 10 > 0$ $\Rightarrow$ minimum distance is

$x = 12 - \frac{24}{7} - \frac{3(18)}{7} = \frac{6}{7}$

$d^2 = \left(\frac{6}{7}\right)^2 + \left(\frac{12}{7}\right)^2 + \left(\frac{18}{7}\right)^2 = \frac{504}{49}$

$d = \sqrt{504/49}$

(R4) Find the min of $x^2+4xy+y^2 = f(x,y)$ subject to $x-y-6=0$

$\nabla f = \lambda \nabla g$ where $g = x-y-6=0$

$\nabla f = \langle 2x+4y,\ 4x+2y \rangle$

$\nabla g = \langle 1, -1 \rangle$

$$\left.\begin{array}{l} 2x+4y=\lambda \\ 4x+2y=-\lambda \end{array}\right\} \quad \left.\begin{array}{l} 6x+6y=0 \\ x-y=6 \end{array}\right\} \quad \begin{array}{l} x=-y \\ -2y=6 \Rightarrow y=-3 \\ x=3 \end{array}$$

$x-y-6=0$

maximum at $(3,-3)$

(R5) evaluate the integral

$$\int_0^1\int_0^2 \frac{y}{1+x^2}\,dy\,dx$$

$$V = \int_0^1 \frac{y^2}{2(1+x^2)}\Big|_0^2 dx = \int_0^1 \frac{2}{1+x^2}\,dx = 2\tan^{-1}(x)\Big|_0^1 = \frac{\pi}{2}$$

(R6) $$\int_0^1\int_0^1 \frac{y}{(xy+1)^2}\,dx\,dy = \int_0^1\int_0^1 y(xy+1)^{-2}\,dx\,dy$$

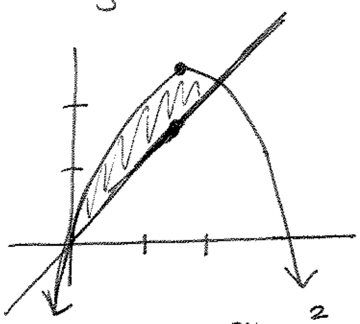
$$V = \int_0^1 -\frac{y(xy+1)^{-1}}{y}\Big|_0^1 dy = \int_0^1\left[\frac{-1}{xy+1}\right]_0^1 dy$$

$$= \int_0^1\left(1-\frac{1}{y+1}\right)dy = 1-\ln 2$$

(R7) $\displaystyle\int_1^2\int_0^{x^2} \frac{y^2}{x}\,dy\,dx$

$$V=\int_1^2 \frac{y^3}{3x}\Big|_0^{x^2} dx = \int_1^2 \frac{x^5}{3}dx = \frac{x^6}{18}\Big|_1^2 = \frac{32}{9}-\frac{1}{18}=\frac{63}{18}$$
$$=3.5$$

(R8) $\displaystyle\iint_S (x^2-xy)\,dA$ S is the region between $y=x$ + $y=3x-x^2$



$x=3x-x^2$

$-2x=-x^2$

$x=2$

$y=3x-x^2$

$y'=-2x+3$

$0 \Rightarrow x=\frac{3}{2}$

max at

$$\int_0^2\int_x^{3x-x^2} x^2-xy\,dy\,dx = \int_0^2 x^2y-\frac{xy^2}{2}\Big|_x^{3x-x^2}dx$$

$$=\int_0^2 3x^3-x^4-x(9x^2-6x^3+x^4)-x^3+\frac{x^3}{2}\,dx$$

$$=\int_0^2 \frac{5}{2}x^3-x^4-9x^3+6x^4-x^5\,dx$$

$$=\left[\frac{5x^4}{8}-\frac{x^5}{5}-\frac{9x^4}{4}+\frac{6x^5}{5}-\frac{x^6}{6}\right]_0^2$$

$$=10-\frac{32}{5}-36+\frac{192}{5}-\frac{32}{3}$$

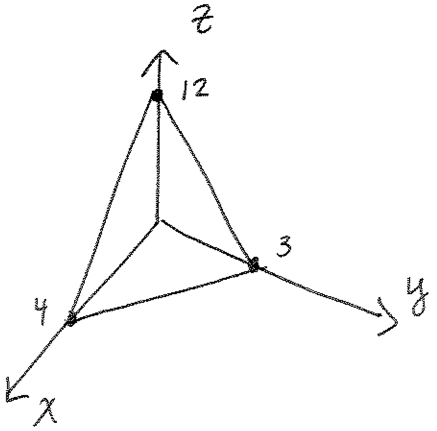
$$=-\frac{8}{15}$$

(R9) Sketch + find volume of tetrahedron bounded by coordinate planes + $3x+4y+z-12=0$

when $x=0 \Rightarrow z=12-4y$

$y=0 \Rightarrow z=12-3x$

$z=0 \Rightarrow 4y=12-3x \Rightarrow y=3-\frac{3}{4}x$



$$\int_0^4 \int_0^{3-\frac{3}{4}x} (12-3x-4y)\,dy\,dx$$

$$= \int_0^4 \left[12y-3xy-\frac{4y^2}{2}\right]_0^{3-\frac{3}{4}x} dx$$

$$= \int_0^4 12\left(3-\frac{3}{4}x\right) - 3x\left(3-\frac{3}{4}x\right) - 2\left(3-\frac{3}{4}x\right)^2 dx$$

$$= \int_0^4 36 - 9x - 9x + \frac{9}{4}x^2 - 18 + 9x - \frac{9}{8}x^2\,dx$$

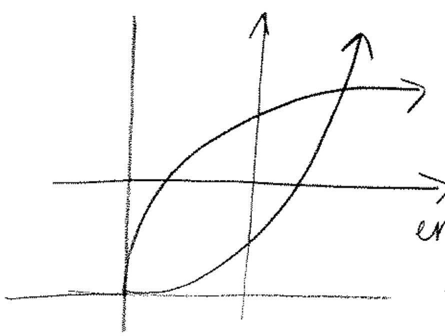
$$= \left[36x - \frac{9x^2}{2} - \frac{9x^2}{2} + \frac{9}{4}\frac{x^3}{3} - 18x + \frac{9x^2}{2} - \frac{9}{8}\frac{x^3}{3}\right]_0^4$$

$$= 144 - (36)4 + 48 - 72 + 72 - 24$$

$$= 24$$

(R10) change the order of integration and rewrite the iterated integral.

a) $\int_0^1 \int_{x^2}^{x^{1/4}} f(x,y)\,dy\,dx$



$y = x^2$

$y = x^{1/4}$

enters at $y = x^{1/4} \Rightarrow x = y^4$ $\qquad y: 0 \to 1$

exits at $y = x^2 \Rightarrow x = \sqrt{y}$

new integral $\int_0^1 \int_{y^4}^{\sqrt{y}} f(x,y)\,dx\,dy$