(R1) Equation of plane through $(6,1,0)$, $(5,-2,-1)$, and $(1,-3,4)$

$$\vec{u}_1 = (5-6)\vec{i} + (-2-1)\vec{j} + (-1-0)\vec{k}$$
$$= -\vec{i} - 3\vec{j} - \vec{k}$$
$$\vec{u}_2 = (1-5)\vec{i} + (-3+2)\vec{j} + (4+1)\vec{k}$$
$$= -4\vec{i} - \vec{j} + 5\vec{k}$$
$$\vec{n} = \vec{u}_1 \times \vec{u}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & -1 \\ -4 & -1 & 5 \end{vmatrix} = (-15-1)\vec{i} - (-5-4)\vec{j} + (1-12)\vec{k}$$

$$= -16\vec{i} + 9\vec{j} - 11\vec{k} \quad \text{w/ } (6,1,0)$$
$$-16(x-6) + 9(y-1) - 11z = 0$$
$$-16x + 9y - 11z = -87 \quad \text{OR} \quad \boxed{16x - 9y + 11z = 87}$$

(R2) a) find the velocity and accel vector at $t=0$ if
$$\vec{r}(t) = \sec t\,\vec{i} + \tan t\,\vec{j}$$
$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \sec t \tan t\,\vec{i} + \sec^2 t\,\vec{j}$$
$$\vec{v}(0) = \vec{j}$$
$$\vec{a}(t) = \sec^3 t + \sec t \tan^2 t\,\vec{i} + 2\sec^2 t \tan t\,\vec{j}$$
$$\vec{a}(0) = \vec{i}$$

b) find the position vector if $\vec{v}(t) = (t^2+1)\vec{i} - t\vec{j}$ and $\vec{r}(0) = 2\vec{i} - \vec{j}$

Since $\vec{v} = \dfrac{d\vec{r}}{dt} \Rightarrow r(t) = \left(\dfrac{t^3}{3} + t + c_1\right)\vec{i} - \left(\dfrac{t^2}{2} + c_2\right)\vec{j}$

if $\vec{r}(0) = 2\vec{i} - \vec{j}$ then $c_1 = 2$, $c_2 = 1$ so that

$$\vec{r}(t) = \left(\frac{t^3}{3} + t + 2\right)\vec{i} - \left(\frac{t^2}{2} + 1\right)\vec{j}$$

R3 a) write the symmetric equations for a line through $(0,2,3)$ and $(2,-4,-1)$.

direction vector: $\langle 2-0, -4-2, -1-3\rangle = \langle 2,-6,-4\rangle$

use the point $(0,2,3)$:

$x = 0 + 2t \quad y = 2 - 6t \quad z = 3 - 4t$

$$\Rightarrow \quad \frac{x}{2} = \frac{y-2}{-6} = \frac{z-3}{-4}$$

or $x = 2+2t,\ y = -4-6t,\ z = -1-4t$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y+4}{-6} = \frac{z+1}{-4}$$

Same as $\dfrac{x-2}{1} = \dfrac{y+4}{-3} = \dfrac{z+1}{-2}$ scaled direction vector by factor of 2.

b) find symmetric equations of a line through $(4,5,8)$ and perp to the plane $3x + 5y + 2z = 30$

normal to plane $\Rightarrow \vec{n} = 3\vec{i} + 5\vec{j} + 2\vec{k}$

$\vec{n}$ = direction vector so

$x = 4 + 3t,\ y = 5 + 5t,\ z = 8 + 2t$

$$\boxed{\frac{x-4}{3} = \frac{y-5}{5} = \frac{z-8}{2}}$$

(R4) Find the unit normal to the curve and the binormal

$\vec{r}(t) = \sin 3t\,\vec{i} + \cos 3t\,\vec{j}$

$$\vec{T} = \frac{\vec{r}\,'(t)}{\|r'(t)\|} \qquad \vec{N} = \frac{\vec{T}\,'(t)}{\|T'(t)\|}$$

$\vec{r}\,'(t) = 3\cos 3t\,\vec{i} - 3\sin 3t\,\vec{j}$

$\|\vec{r}\,'(t)\| = \sqrt{3^2(\cos^2 3t + \sin^2 3t)} = 3$

$\boxed{\vec{T} = \cos 3t\,\vec{i} - \sin 3t\,\vec{j}}$

$\vec{T}\,'(t) = -3\sin 3t\,\vec{i} - 3\cos 3t\,\vec{j}$

$\|\vec{T}\,'(t)\| = 3$

$\boxed{\vec{N} = -\sin 3t\,\vec{i} - \cos 3t\,\vec{j}}$

$\vec{B} = \vec{T} \times \vec{N}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos 3t & -\sin 3t & 0 \\ -\sin 3t & -\cos 3t & 0 \end{vmatrix} = 0\vec{i} + 0\vec{j} + (-\cos^2 3t - \sin^2 3t)\vec{k}$$

$$= -\vec{k}$$

$\boxed{\vec{B} = -\vec{k}}$

(R5) Sketch the level curves for $z = x^2 + y$ for $k = -4, -1, 0, 1, 4$

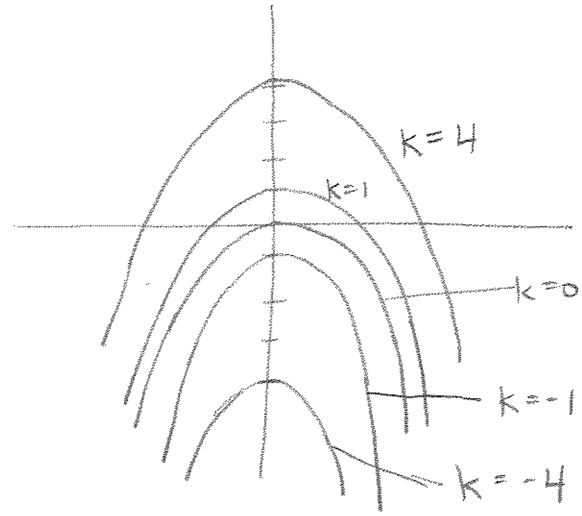
$-4 = x^2 + y \Rightarrow y = -x^2 - 4$

$-1 = x^2 + y \Rightarrow y = -x^2 - 1$

$k = 0 \Rightarrow y = -x^2$

$1 = x^2 + y \Rightarrow y = -x^2 + 1$

$4 = x^2 + y \Rightarrow y = -x^2 + 4$




(R6) a) verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ for $f(x,y) = 2x^2y^3 - x^3y^5$

$\frac{\partial f}{\partial x} = 4xy^3 - 3x^2y^5$ $\qquad \frac{\partial f}{\partial y} = 6x^2y^2 - 5x^3y^4$

$\frac{\partial^2 f}{\partial x \partial y} = 12xy^2 - 15x^2y^4 \Leftrightarrow \frac{\partial^2 f}{\partial x \partial y} = 12xy^2 - 15x^2y^4$ ✓

b) verify that $f(x,y) = \ln(4x^2 + 4y^2)$ is harmonic $(f_{xx} + f_{yy}) = 0$

$f_x = \frac{8x}{4(x^2+y^2)} = \frac{2x}{x^2+y^2}$ $\qquad f_y = \frac{2y}{x^2+y^2}$

$f_{xx} = \frac{2x^2 + 2y^2 - 2x(2x)}{(x^2+y^2)^2}$ $\qquad f_{yy} = \frac{2x^2 + 2y^2 - 4y^2}{(x^2+y^2)^2}$

$= \frac{2y^2 - 2x^2}{(x^2+y^2)^2}$ $\qquad = \frac{2x^2 - 2y^2}{(x^2+y^2)^2}$

$f_{xx} + f_{yy} = \frac{2y^2 - 2x^2 + 2x^2 - 2y^2}{(x^2+y^2)^2} = 0$ ✓

(R7) find the indicated limit or state that it DNE.

a) $\lim_{(x,y)\to(0,0)} \frac{xy}{(x^2+y^2)^2} \to \frac{0}{0}$ try polar...

in polar: $\lim_{r\to 0} \frac{r^2\cos\theta\sin\theta}{(r^2)^2} = \frac{\cos\theta\sin\theta}{r^2} \Rightarrow$ DNE

b) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

along the line $x=y^2 \Rightarrow \lim_{(x,y)\to(0,0)} \frac{y^4}{2y^4} = \frac{1}{2}$

along the line $x=2y^2 \Rightarrow \lim_{(x,y)\to(0,0)} \frac{2y^4}{4y^4+y^4} = \frac{2}{5}$

different limits. $\Rightarrow$ DNE

(R8) a) find $\nabla f$ for $f(x,y) = xz\ln(x+y+z)$

$\nabla f = \left\langle \frac{xz}{x+y+z} + z\ln(x+y+z),\ \frac{xz}{x+y+z},\ \frac{xz}{x+y+z} + x\ln(x+y+z) \right\rangle$

b) find the equation of the tangent plane for the surface $x^2+2xy-y^2$ at $(1,-2,-7)$

$z = f(1,-2,-7) + \nabla f(1,-2,-7)\cdot\langle x-1, y+2, z+7\rangle$

$f(1,-2,-7) = 1-4-4 = -7$

$\nabla f = \langle 2x+2y,\ 2x-2y,\ 0\rangle \qquad \nabla f(1,-2,-7) = \langle -2, 6, 0\rangle$

$z = -7 + \langle -2,6,0\rangle\langle x-1, y+2, z+7\rangle$

$\Rightarrow \quad z+7 = -2x+2+6y+12 \Rightarrow \boxed{-2x+6y-z=-7}$

R9 a) Find the directional derivative of $f$ at $\vec{p}$ in direction $\vec{a}$

$f(x,y) = e^{-xy}$ $\quad \vec{p} = (1,-1)$ $\quad \vec{a} = -\vec{i} + \sqrt{3}\,\vec{j}$

$$\vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = -\frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$$

$$D_{\vec{u}} f(\vec{p}) = \nabla f(\vec{p}) \cdot \vec{u}$$

$$\nabla f = \langle -ye^{-xy}, -xe^{-xy} \rangle$$

$$\nabla f(1,-1) = \langle e, -e \rangle$$

$$D_{\vec{u}} f = \langle e, -e \rangle \cdot \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

$$= \frac{e}{2} - \frac{\sqrt{3}e}{2} = \frac{e}{2}(1 - \sqrt{3})$$

b) In what direction does $f(x,y,z) = xy + yz + xz$ increase most rapidly at $(1,2,3)$? What is the rate of change?

the direction is $\nabla f$, rate of change is $\|\nabla f\|$

$$\nabla f = \langle y+z, x+z, x+y \rangle$$

$\nabla f(1,2,3) = \langle 5, 4, 3 \rangle \Rightarrow$ direction $5\vec{i} + 4\vec{j} + 3\vec{k}$

rate of change is $\sqrt{5^2 + 4^2 + 3^2} = \sqrt{50}$