MATHEMATICS 3100. Second Midterm Test: Solutions.

March 13, 2003

The exam is "closed book, closed notes".

1. [20 points] State and prove the concurrence theorem for the perpendicular bisectors of a triangle.

Solution.

Theorem: All three perpendicular bisectors of a triangle cross in a common point.

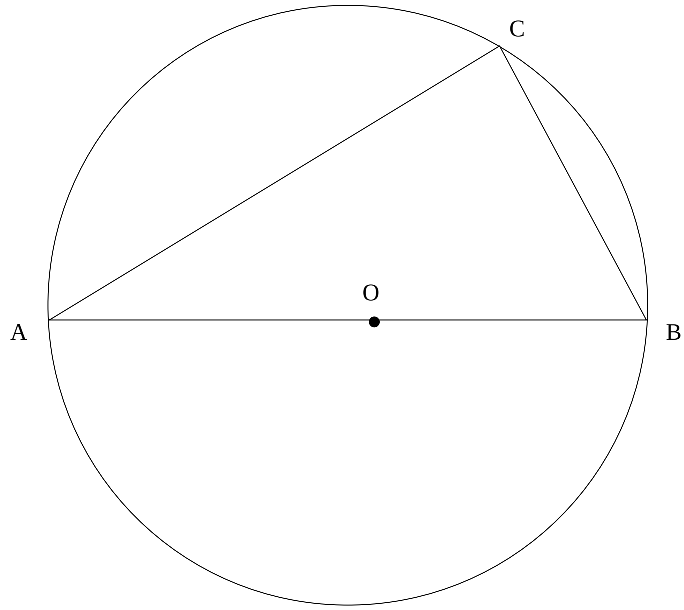
Proof: Recall that a perpendicular bisector of a segment AB consists of points which are within the same distance from A and from B. Thus, if L is the perp. bisector of the side AB in the triangle ΔABC and L' is the perp. bisector of the side BC and O is the point of intersection of L and L' then

$$|OA| = |OB| = |OC|.$$

Thus $|OA| = |OC|$ and therefore O lies on the perp. bisector of the side CA. Hence all three perp. bisectors cross at the point O. $\square$

2. [20 points] Consider the triangle ABC in the figure below inscribed in the circle where AB is a diameter. Compute the angle $\angle ACB$. Justify your solution!

Solution. By the basic theorem about inscribed and the central angle, $\angle ACB = \frac{1}{2}\angle AOB$. However $\angle AOB = \pi$, hence $\angle ACB = \pi/2$, is the right angle. $\square$

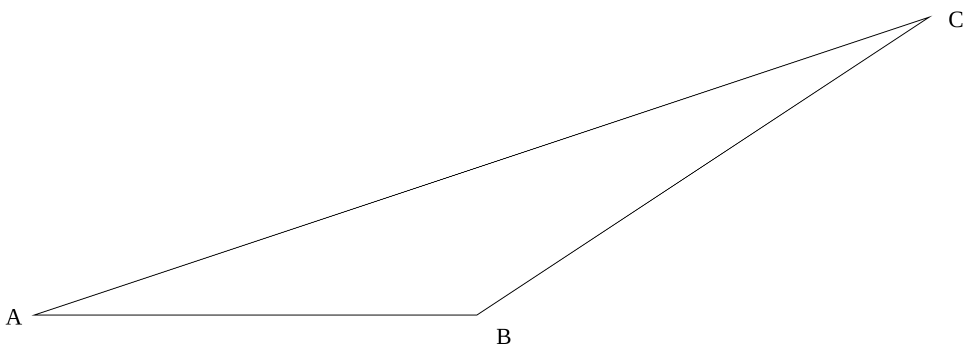


3. [20 points] Given segment of unit length in the plane (figure below), construct (using compass and ruler) a segment of the length 2/3.

Solution. See the textbook. □

4. [20 points] Using compass and ruler construct all three altitides in the given triangle:

Solution. See the textbook. □



5. [20 points] State and prove the law of sines in a triangle.

Solution. See the textbook or the lecture notes. □